2026

GENERAL MATHEMATICS

Level 3

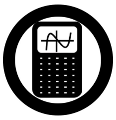
SEQUENCES

General Mathematics: Level 3

GM3 - SEQUENCES

By Jess Bertram

With sincere thanks to John Short and Rick Smith.

ICON:	MEANING:
	Worked example
	Complete with your teacher
	Try it yourself
	CAS Calculator can be used
	Tips / shortcuts

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GENERAL MATHEMATICS - LEVEL 3

GM3 SEQUENCES

BY JESS BERTRAM

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Part 1 INTRO TO SEQUENCES

INTRODUCTION TO SEQUENCES

A sequence is a list of numbers in a specific order that follow a pattern.

Examples of sequences

2, 5, 8, 11, 14, … ← Add 3 each time
100, 50, 25, 12.5, … ← Divide by 2 each time (or multiply by 0.5)
50, 55, 60.5, 66.55, … ← Increase by 10% each time

Why are sequences useful?

Sequences are used to model change over time.

They help us answer questions like:

- How fast is something growing or shrinking?
- What will happen in the future?
- What will the total be after many steps?
- Will the values get very large, very small, or level off?

Each step in a sequence can represent a moment in time. Think of:

- Day 1, Day 2, Day 3…
- Year 1, Year 2, Year 3…
- Measurement 1, 2, 3…

At each step, something changes by a set amount.

That pattern is the rule of the sequence.

If you can describe how the numbers change, you have a sequence.

This workbook covers 3 different types of sequences.

1. Arithmetic sequences
2. Geometric sequences
3. Combination relations

We'll begin with simple number patterns and gradually build towards powerful mathematical models used to describe growth, decay, and change in the real world.

ARITHMETIC VS GEOMETRIC SEQUENCES

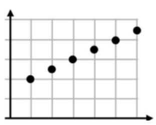
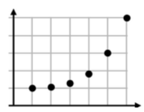
Not all sequences change in the same way.

There are two main types of patterns we will learn:

1. Arithmetic sequences
2. Geometric sequences

Arithmetic sequences	Geometric sequences
Add or subtract the same amount Example: 2, 5, 8, 11, 14, … (+3) The change (+3) is known as a common difference (d) This creates linear change (straight-line pattern).	**Multiply** by the same amount Example: 25, 50, 100, 200, … (×2) The change (×2) is known as a common ratio (r) This creates exponential change (curved pattern).

Side by side comparison:

Feature	**Arithmetic Sequence**	**Geometric Sequence**
How it changes	Add or subtract a constant	Multiply by a constant
Example	4, 9, 14, 19, 24,… (+5)	1, 4, 16, 64, 256, … (× 4)
Change name	Common difference (d)	Common ratio (r)
Growth type	Linear	Exponential
Graph shape	Straight line	Curve
Example graph		

With your teacher.

Identify the pattern in each sequence and mark as arithmetic (A) or geometric (G).

a) 4, 6, 8, 10, 12… b) 5, 10, 15, 20, 25… c) 10, 20, 40, 80, 160,…

d) 1, 3, 9, 27, 82,… e) 9, 18, 27, 36, 45,… f) 3, 6, 12, 24, 48,…

Part 2 ARITHMETIC SEQUENCES

WHAT IS AN ARITHMETIC SEQUENCE?

An arithmetic sequence is a pattern where the numbers change by the same amount each step. This amount can be:

Positive	(increasing sequence)	→ 4, 7, 10, 13, 16, …	(+3 each time)
Negative	(decreasing sequence)	→ 15, 10, 5, 0, -5 …	(–5 each time)
Zero	(constant sequence)	→ 8, 8, 8, 8, …	(no change, +0)

Arithmetic sequences add or subtract the SAME number each time.

Arithmetic sequences model situations where something increases or decreases by a fixed amount over time. These sequences can represent real life situations.

Saving $50 every week	→ 50, 100, 150, 200, …
Temperature drops 2°C each hour	→ 18, 16, 14, 12, …
A plant grows 3 cm each day	→ 5, 8, 11, 14, …
You walk 1000 extra steps each day	→ 3000, 4000, 5000, 6000, …

Important terminology

Terminology	Meaning	Example: 4, 7, 10, 13, …
Term	A value in the sequence	7 is the 2nd term
First term (t_1)	The starting value	t_1 = 4
Common difference (d)	The amount added/subtracted	d = +3
n	The position of a term	1st , 2nd , 3rd …
$\boldsymbol{t_n}$	The value of the nth term	t_1 = 4, t_2 = 7…

You can represent terms with subscripts:

t_1 = first term, t_2 = second term, t_3 = third term, t_{99} = 99th term, t_n = nth term

Remember: If a sequence changes by adding or subtracting the same amount each time, it is an arithmetic sequence.

With your teacher.

1. Identify if each sequence is arithmetic. If it is, state the common difference (d).

		Arithmetic sequence?	Common difference (d)
a.	3, 6, 9, 12, …		
b.	10, 7, 4, 1, …		
c.	5, 10, 20, 40, …		
d.	5, 5, 5, 5, …		
e.	2, 5, 9, 14, …		
f.	100, 90, 80, 70, …		

2. Write the first 5 terms (t_1, t_2, t_3, t_4, t_5) of each arithmetic sequence.
   a. Start at 4, add 6 each time

   b. Start at 20, subtract 3 each time

   c. Start at –2, add 5 each time

   d. Start at 50, no change

With your teacher.

3. Write the first 5 terms (t_1, t_2, t_3, t_4, t_5) for each.
   a. You save $25 every week, starting with $0.

   b. A car loses $1500 in value each year, starting at $18 000.

   c. A plant grows 2 cm per day, starting at 5 cm.

   d. A timer counts down by 10 seconds each minute, starting at 120 seconds.

4. Find the common difference (d), then identify the 1st term (t_1) and the 3rd term (t_3).

		Common difference (d)	1st term (t_1)	3rd term (t_3)
a.	12, 15, 18, 21, …			
b.	40, 35, 30, 25, …			
c.	–5, –2, 1, 4, …			
d.	7, 7, 7, 7, …			
e.	2.5, 5, 7.5, 10, …			
f.	100, 80, 60, 40, …			

Try it yourself (INTRO TO SEQUENCES) **Answers page 77**

1. State whether each of the following sequences are arithmetic. If they are, state the common difference (d), first term (t_1) and the third term (t_3)
   a. 9, 12, 15, 18, …
   b. 16, 8, 4, 2, …
   c. 5, 3, 1, –1, –3 …
   d. 2, 5, 9, 14, …
   e. –10, –5, 0, 5, …
   f. 7, 7, 7, 7, …

2. Fill in the missing terms.
   a. 2, __, __, 8, 10
   b. __, 14, 11, 8, __
   c. –3, __, 1, __, 5
   d. 50, 45, __, 35, __

3. Identify the first term (t_1), then write the next 5 terms in each sequence (6 total).
   a. A phone plan costs $30 in the first month and increases by $2 per month.
   b. A tank loses 5 litres of water each hour, starting at 40 litres.
   c. A runner improves her time by 0.4 seconds each race, starting at 15.0 seconds.

4. Create your own sequence of 5 terms, clearly identifying the first term (t_1) and common difference (d) in each.
   a. Write any arithmetic sequence that shows growth (increasing).
   b. Write any arithmetic sequence that shows decay (decreasing).
   c. Write an arithmetic sequence that includes a negative number.
   d. Write an arithmetic sequence whose first term is 0.
   e. Write a constant arithmetic sequence.

5. A sequence starts at 12 and every term is 7 less than the one before.
   a. Write the first 6 terms.
   b. Is this growth or decay?
   c. What is the common difference (d)?
   d. Will this sequence eventually become negative?
   e. What will the 10th term be?

TABLES AND GRAPHS

We now know that an arithmetic sequence changes by adding or subtracting the same amount each time.

To understand them more clearly visually, we can show them in tables and graphs.

Turning sequences into tables

Make a column (or row) for each value you want to record.

Standard columns would be term position (n) and value of each term (t_n)

Worked example. From sequence → table

Turn the following sequence into a table: 4, 7, 10, 13, 16, …

We list the term position (n) and the value of the term (t_n):

Term position (n)	**Value (t_n)**
1	4
2	7
3	10
4	13
5	16

With your teacher. Create a table for the following sequences.

a. 5, 9, 13, 17, 21, 25, 29, 33, 37

Term position (n)	Value (t_n)
1	5
2	9
3	
4	
5	
6	
7	
8	
9	

b. 40, 36, 32, 28, 24, 20, 16, 12, 8

Term position (n)	Value (t_n)

Turning sequences into graphs

1. Make a table of term position (n), term value (t_n), and points (n, t_n)
2. Plot the points (n, t_n)

Worked example. From sequence → graph

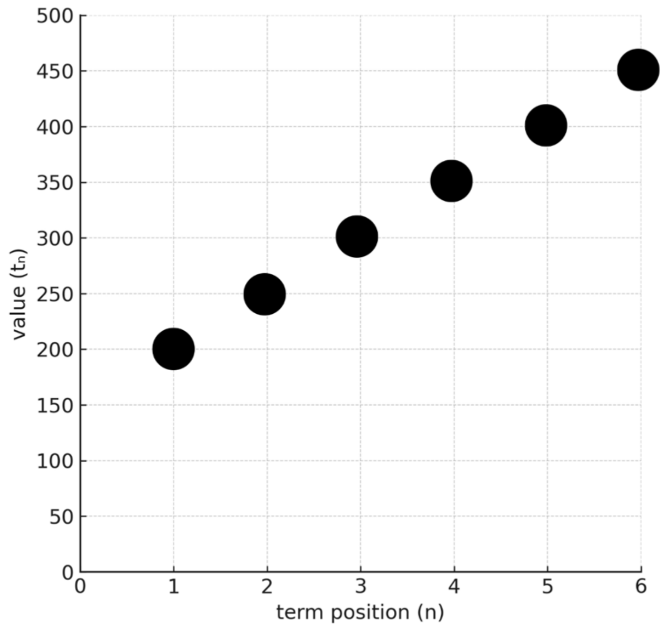
Jasmin starts a new job and earns $200 in her first week. Each week, she earns $50 more than the previous week as she takes on extra responsibilities.

This sequence could be written as: 200, 250, 300, 350, 400, 450, …

Step 1: Make a table of term position (n) and term value (t_n) for the first 6 terms.

Term position (n)	Value (t_n)		Point @ (n, t_n)
1	200	→	(1, 200)
2	250	→	(2, 250)
3	300	→	(3, 300)
4	350	→	(4, 350)
5	400	→	(5, 400)
6	450	→	(6, 450)

Step 2: Plot the points. We only plot the points (discrete values). We do not join them with a continuous line because sequences only exist at set intervals.



When we plot arithmetic sequences on a graph, they form a straight-line pattern.

With your teacher.

A radioactive sample decays by 12 grams every hour. At the start of the monitoring period the sample has 100 grams of material remaining.

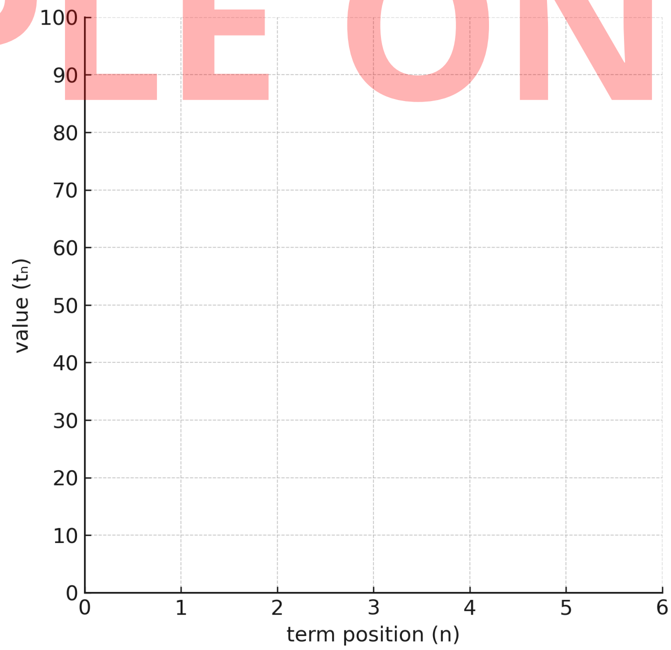
a. Represent this situation using a sequence. Write the first 6 terms ($t_1 \rightarrow t_6$)

b. Identify the first term (t_1) and common difference (d).

c. Fill in the below table.

Term position (n)	Value (t_n)		Point @ (n, t_n)
		$\rightarrow$	
		$\rightarrow$	
		$\rightarrow$	
		$\rightarrow$	
		$\rightarrow$	
		$\rightarrow$	

d. Graph your points.



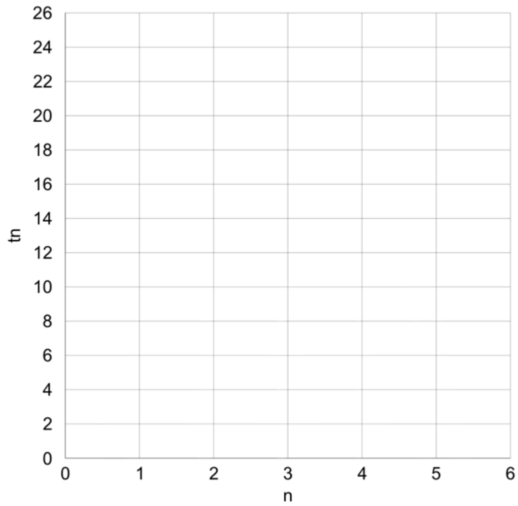
e. If we look at the 10th term, the sample has a weight of -8 grams. Is this possible in real life? Discuss.

Try it yourself (TABLES AND GRAPHS - ARITHEMTIC) **Ans pg77**

1. Write each sequence as a table (n, t_n), then plot the first 6 terms of each sequence on a graph.

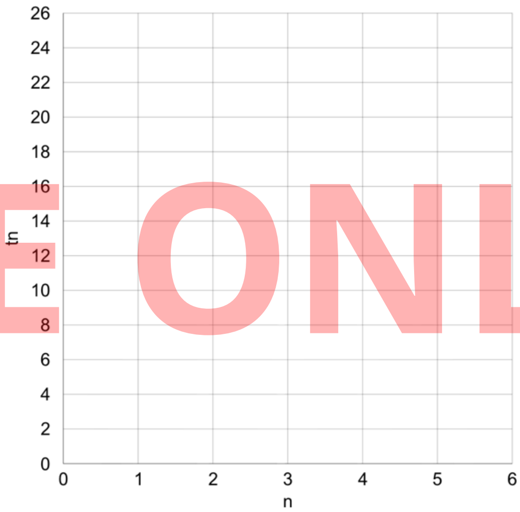
   a. 5, 9, 13, …

n	t_n		(n, t_n)
		→	
		→	
		→	
		→	
		→	
		→	



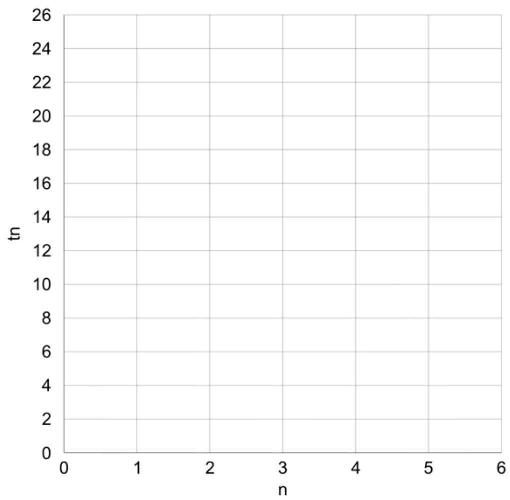
   b. 12, 10, 8, …

n	t_n		(n, t_n)
		→	
		→	
		→	
		→	
		→	
		→	



   c. 4, 4, 4, …

n	t_n		(n, t_n)
		→	
		→	
		→	
		→	
		→	
		→	

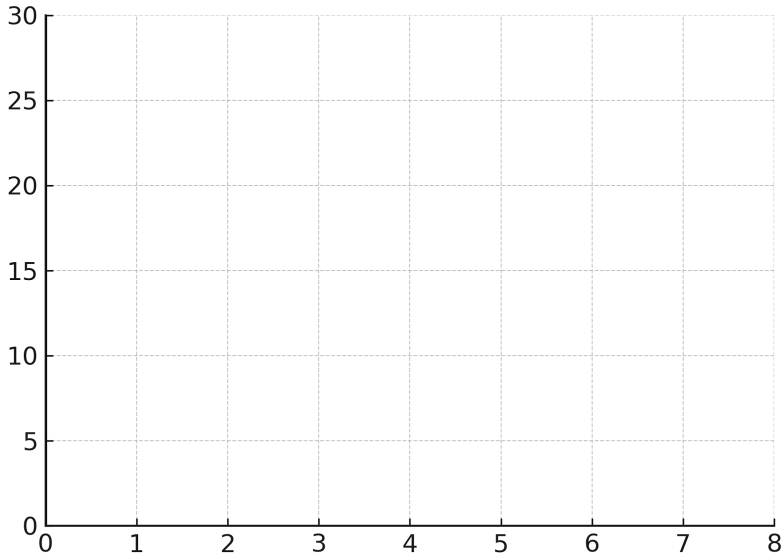


2. Answer the following questions for each graph in question 1:
   a. Is it increasing (growth), decreasing (decay), or constant?
   b. Does it form a straight-line pattern?
   c. What is the common difference (d)?

Try it yourself (TABLES AND GRAPHS - ARITHMETIC cont) **Ans pg77**

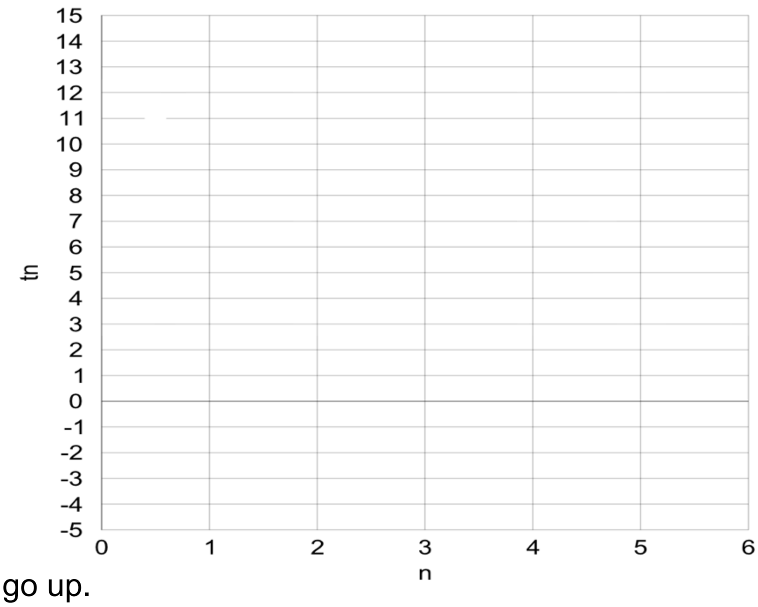
3. For the sequence: 7, 10, 13, 16, 19, …
   a. Identify the common difference (d)
   b. Fill in the details in the table below and plot the first 8 terms.

n	t_n		(n, t_n)
		→	
		→	
		→	
		→	
		→	
		→	
		→	
		→	



4. A sequence has $t_1 = 15$ and $d = -4$.
   a. Fill in the table with the first 6 terms.
   b. Graph the first 6 terms.
   c. Does the graph show growth or decay?

n	t_n		(n, t_n)
		→	
		→	
		→	
		→	
		→	
		→	



5. True or False? Explain.
   a. All arithmetic sequences go up.
   b. An arithmetic sequence can have negative numbers.
   c. Arithmetic sequences always make a straight-line pattern on a graph.

NTH TERM RULE (EXPLICIT RULE)

So far, we've looked at recursion, where we find each term by adding the common difference to the previous term.

Example of recursion.

A bus service in Port Huon charges \$2.00 for a day pass. The managers decide to increase this charge by \$0.50 per year to help cover some costs.

We know that:

t_1 = \$2.00

d = \$0.50

So the sequence for the first 6 years would be:

\$2.00, \$2.50, \$3.00, \$3.50, \$4.00, \$4.50, …

This shows us the value from t_1 to t_6, but what if we wanted to know t_{100}? We would have to write out 100 terms.

That's where the **nth term rule** comes in, it allows us to jump to any term.

Recursion	nth term rule
Use the previous term to find the next term. Example: $t_1 = 2$ ← the first term $d = 0.5$ ← the common difference → 2, 2.5, 3, 3.5, 4, … **Good for:** ✅ Building the sequence step by step ✅ Real-world situations ✅ Using calculators or spreadsheets ❌ Hard to jump to t_{100}	**Jump straight to any term.** **No need for previous terms.** *Also called explicit / sequence rule* Example: t_n = 2 + (n − 1) × 0.5 t_{100} = 2 + (100 − 1) × 0.5 t_{100} = 54.50 **Good for:** ✅ Predicting far-off terms ✅ Solving problems quickly ✅ Finding the value of a certain term ❌ Doesn't show the pattern as clearly

Nth term rule (arithmetic sequences)

$$t_n = a + (n-1)d$$

Where:

t_n= Value of the term in n position

a = The first term (same as t_1)

n = Term position. How far along the sequence the term is.

d = Common difference. The amount added or subtracted each time.

Worked example.

A tree is planted in a garden when it is 10cm tall and grows 2cm every day. Write a rule for this arithmetic sequence.

$t_n = a + (n-1)d$

$a \quad = \quad 10$

$d \quad = \quad +2$

$\therefore\ t_n = 10 + (n-1)2$ ← this is the nth term rule

If we wanted to find the value after a year (365 days) we could use the formula.

$t_n = 10 + (n-1)2$

$t_{365} = 10 + (365-1)2$

$t_{365} = 738cm$

The plant would be 738cm (7.38m) in 1 year.

What about in 2 years' time (730 days)?

$t_{730} = 10 + (730-1)2$ → The plant would be 1,468cm or 1.468m tall.

Where does the nth term rule come from?

Each term in a sequence is built by adding the common difference (d) to the term before it. Starting with the first term (a), we could write this as:

$$a, \qquad a+d, \qquad a+2d, \qquad a+3d, \qquad \ldots a+(n-1)d$$

This pattern shows how the sequence grows step by step. The nth term is made by starting at a and adding (n−1) lots of d, which gives the nth term rule.

With your teacher.

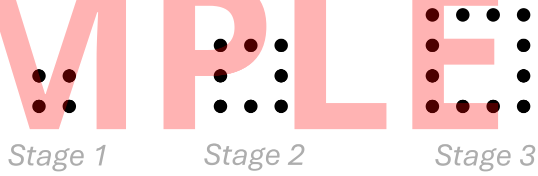
1. A water tank is leaking. A mechanic checks the tank every hour and finds that the tank has lost 5 litres of water each check.

   a. Write the sequence rule if the tank had 80L in it when the first check was made.

   b. Find the volume water in the tank at the 12-hour check.

   c. Find the volume of water in the tank at the 24-hour check. Discuss.

2. Consider the first 3 stages of the following pattern.



   a. Draw the next two stages of the pattern.

   b. Detail the first 6 terms of the arithmetic sequence formed by the number of dots involved.

   c. Find a rule for the pattern. (nth term rule)

   d. Find the number of dots involved in the 20th stage of the pattern.

   e. 224 is a term in this arithmetic sequence. In what position is it?
   (Hint: set 224 as t_n then solve for n)

The Nth term rule can be found from any 2 terms.

Step 1: Use both terms to find the common difference (d)

Step 2: Use common difference (d) to find the first term (a)

Step 3: Sub (d) and (a) into the nth term rule

Worked example.

The 12th term of an arithmetic sequence is 28. The 18th term is 46. Find;

a. The common difference 'd' and first term 'a'.

b. The rule for the sequence.

c. The 31st term.

To find 'd'

$$\begin{aligned} t_{12} + 6d &= t_{18} \\ 28 + 6d &= 46 \\ 6d &= 18 \\ d &= 3 \end{aligned}$$

← The 12th term, plus 6 lots of the common difference, will equal the 18th term.

To find 'a'

$$\begin{aligned} a + (12 - 1)d &= 28 \\ a + (11)d &= 28 \\ a + (11)3 &= 28 \\ a + 33 &= 28 \\ a &= -5 \end{aligned}$$

← This has been done with the 12th term (28), but we could use the 18th term (46) instead.

Once we've found 'd' and 'a' we can **find the rule.**

$$t_n = a + (n-1)d$$

$$t_n = -5 + (n-1)3$$ ← Nth term rule for the sequence

The rule can be simplified.

$$\begin{aligned} t_n &= -5 + (n-1)3 \\ t_n &= -5 + 3n - 3 \\ t_n &= 3n - 8 \end{aligned}$$

← Simplified rule for the sequence

Note: Simplifying makes the formula faster to use, but hides the relationship between d, a, and n, which is needed for some questions.

Find the 31st term.

$$\begin{aligned} t_{31} &= -5 + (31-1)3 \\ t_{31} &= 85 \end{aligned}$$

← 31st term is 85

With your teacher.

1. The first term of an arithmetic sequence is 14. The 7^{th} term is 50. Find:
    a. The common difference ‘d’ and first term ‘a’.
    b. The rule for the sequence.

2. The 6^{th} term of an arithmetic sequence is 123. The 11^{th} term is 98. Find;
    a. The common difference ‘d’ and first term ‘a’.
    b. The rule for the sequence.
    c. The 16^{th} term.

Try it yourself (NTH TERM RULE - ARITHMETIC) **Ans pg77**

1. For each of the following arithmetic sequences identify the first term ‘a’ and the common difference ‘d’ then write the nth term rule for the sequence.
    a. 3, 8, 13, 18, …
    b. -7, -4, -1, 2, …
    c. 13, 25, 37, 49, …
    d. 10, 8, 6, 4, …
    e. 30, 40, 50, 60, …
    f. 5.0, 5.5, 6.0, 6.5, …
    g. -4, -8, -12, -16, …
    h. 104, 110, 116, 122, …

2. For the sequence 5, 9, 13, 17, …
    a. Write out the sequence until the eighth term.
    b. Find the rule for the sequence.
    c. Use your rule to find the 8^{th} term of the sequence to verify your sequence in part (a).

3. For the sequence 15, 11, 7, 3, …
    a. Write out the sequence until the tenth term.
    b. Find the rule for the sequence. (ie Find an expression for the n^{th} term)
    c. Use your rule to find the 10^{th} term of the sequence to verify your sequence in part (a).

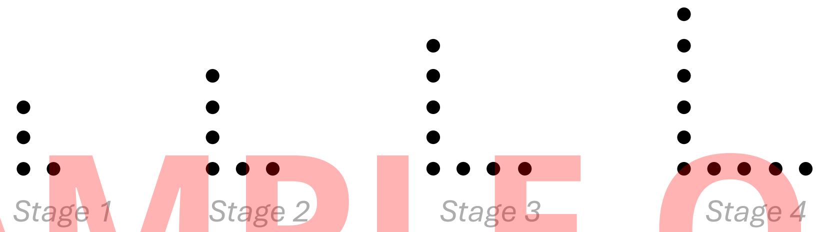
4. For the sequence -108, -104, -100, -96, …
    a. Write out the sequence until the seventh term.
    b. Find an expression for the n^{th} term
    c. Use your expression to find the 7^{h} term of the sequence to verify your sequence in part (a).

5. Find the nth term of these sequences:
    a. Find the 20^{th} term of the sequence: 10, 8, 6, 4, …
    b. Find the 10^{th} term of the sequence: 3, 8, 13, 18, …
    c. Find the 15^{th} term of the sequence: 5, -1, -7, -13, …
    d. Find the 100^{th} term of the sequence: 3.0, 3.2, 3.4, 3.6, …
    e. Find the 52^{th} term of the sequence: 15.0, 15.5, 16.0, 16.5, …
    f. Find the 17^{th} term of the sequence: 600, 540, 480, 420, …

Try it yourself (NTH TERM RULE - ARITHMETIC cont) **Ans pg77**

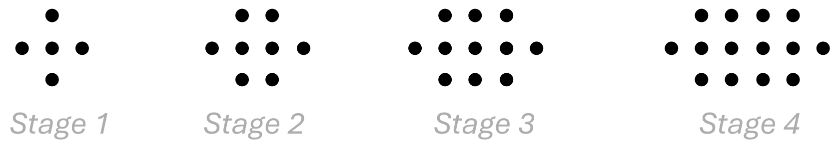
6. Identify the first term (a) and common difference (d) involved in each of the following sequences, then write the first three terms:
   a. On first measurement a lettuce leaf had a weight of 72g but a caterpillar eats 4.5g of it every hour.
   b. A tomato plant has a height of 6cm on week 1 and grows 15cm each week.
   c. A club's membership is growing by 20 each year, starting at 35.
   d. An athlete finds that her sprint time shows a regular improvement of 0.2 seconds each time she races. Her first race sprint time was 13.3 seconds.

7. For the following pattern:



   a. Draw the next two stages of the above pattern.
   b. Detail the first 6 terms of the arithmetic sequence formed by the number of dots involved.
   c. Find a rule for the pattern. (ie Find an expression for the number of dots in the n^{th} stage.)
   d. Find the number of dots involved in the 50^{th} stage of the pattern.

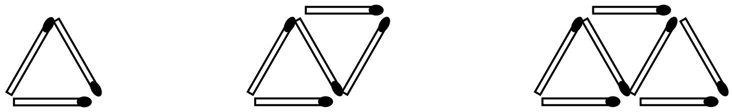
8. For the following pattern:



   a. Draw the next two stages of the pattern.
   b. Detail the first 6 terms of the arithmetic sequence formed by the number of dots involved.
   c. Find a rule for the pattern. (ie Find an expression for the number of dots in the n^{th} stage.)
   d. Find the number of dots used for the 43^{rd} stage of the pattern.

Try it yourself (NTH TERM RULE - ARITHMETIC cont) **Ans pg77**

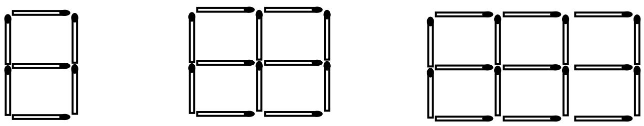
9. For the following pattern:



Stage 1 Stage 2 Stage 3

   a. Draw the next two stages of the pattern.
   b. Detail the first 5 terms of the arithmetic sequence formed by the number of matchsticks involved.
   c. Find a rule for the pattern.
   d. Find the number of matches used for the 100th stage.

10. For the following pattern:



Stage 1 Stage 2 Stage 3

   a. Draw the next two stages of the pattern.
   b. Detail the first 5 terms of the arithmetic sequence formed by the number of matchsticks involved.
   c. Find a rule for the pattern.
   d. Find the number of matches used for the 27th stage.

11. During MONA's Winter Solstice Festival in Hobart, organisers are setting up a long communal dining experience. Rows of trestle tables are placed end to end to create one continuous banquet table, as shown in the diagrams below.

1 table gives seating for 6 2 tables gives seating for 10

   a. Complete the following grid showing how the number of people that can be seated changes with the number of trestle tables used.

Number of tables	1	2	3	4	5	6
Seating	6	10				

   b. Prove that the number of people seated forms an arithmetic sequence and give the rule for the sequence.
   c. Use your rule to find the number of people that can be seated if 23 tables are used.

Try it yourself (NTH TERM RULE – ARITHMETIC cont) **Ans pg77**

12. Find the position of the term in the arithmetic sequence.
    a. 63 is a term of the arithmetic sequence: 7, 11, 15, 19, …..
    b. -15 is a term of the arithmetic sequence: 15, 12, 9, 6, …..
    c. 46 is a term of the arithmetic sequence: -24, -17, -10, …..
    d. 12.25 is a term of the arithmetic sequence: 4.50, 4.75, 5.00….
    e. 331 is a term of the arithmetic sequence: 99, 107, 115,…..

13. The first term of an arithmetic series is 27. The fifth term is 59. Find:
    a. The common difference 'd' and first term 'a'.
    b. The rule for the sequence.
    c. The 18th term.

14. The second term of an arithmetic series is 19. The 9th term is -30. Find:
    a. The common difference 'd' and first term 'a'.
    b. The rule for the sequence.
    c. The 23rd term.

15. The fourth term of an arithmetic series is 33. The 9th term is 63. Find:
    a. The common difference 'd' and first term 'a'.
    b. The rule for the sequence.
    c. The 19th term.

16. The ninth term of an arithmetic series is -14. The 13th term is -24. Find:
    a. The rule for the sequence.
    b. The 18th term.

17. A wristwatch is slowly losing time at a constant rate each day. On day 1, its owner sets it five minutes fast. By day 8, the watch is 16 minutes slow. How many minutes behind the correct time is the watch on day 4?

18. A swimming pool is being topped up with water from a garden hose. Measurements of the depth of the water are made at regular intervals. The 5th time a measurement was made the water was 72 cm deep. The 12th measurement found that the water was 128 cm deep. Find:
    a. An expression for the depth upon making the nth measurement.
    b. The depth at the time of the first measurement.
    c. The depth upon making the 9th measurement.

SUM OF TERMS (SERIES)

Sum of terms is when we add together the terms in a sequence. It is also known as a series, or an arithmetic series (you will learn about geometric series later).

We can find the sum of any length of sequence, it might be the first 4 terms (S_4), the first 40 terms (S_{40}), or the first 400 terms (S_{400}).

Sequence	**Series (sum)**
List terms in order	Add terms together
2, 4, 6, 8, …	2 + 4 + 6 + 8

Worked example.

Find the sum of 2, 4, 6, 8, … (to 10 terms)

$$S_{10} = 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20$$

$$S_{10} = 110$$

With your teacher.

Find the sum of 20, 16, 12, 8, … (to 12 terms)

$$S_{12} =$$

What if you were asked to calculate a series to 1,000 terms?

As the great philosopher Kimberly "Sweet Brown" Wilkins once said - ain't nobody got time for that. Thankfully mathematicians developed formulas to calculate the sum of any arithmetic series quickly - without having to add every term one by one.

<table>
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$$\boldsymbol{S_n = \frac{n}{2}(a + l)}$$

Where:

a = first term (t_1)

l = last term

n = number of terms

When to use:

When you know the first term and the last term.

</td>
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$$\boldsymbol{S_n = \frac{n}{2}(2a + (n - 1)d)}$$

Where:

a = first term (t_1)

n = number of terms

d = common difference

When to use:

When you know the first term and the common difference.

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Worked examples.

1. The first term of an arithmetic sequence is 6. The 40th term is 123. Find the sum of the first 40 terms.

   We have the first term ($a = 6$) and the last term ($l = 123$).

   $$S_n = \frac{n}{2}(a + l)$$

   $$S_{40} = \frac{40}{2}(6 + 123)$$

   $$S_{40} = 2{,}580$$

2. Find the sum of the first 25 terms of the sequence 100, 95, 90, …

   Given first term ($a = 100$) and can find the common difference ($d = -5$).

   $$S_n = \frac{n}{2}(2a + (n - 1)d)$$

   $$S_{25} = \frac{25}{2}(2(100) + (25 - 1)(-5))$$

   ← Make sure to put extra brackets around any negative terms. Otherwise, in this example, you would accidentally subtract 5 instead of multiplying by -5

   $$S_{25} = 1{,}000$$

With your teacher.

1. An arithmetic sequence starts at 95 and decreases by the same amount each term. The 30th term is 8. Find the sum of the first 30 terms.

2. Find the sum of 14, 18, 22, 26, … for the first 50 terms.

The Casio Classpad calculator can be used to:

- Find the nth term rule given the value and position of 2 terms.
- Find the value of the nth term given the rule

Finding the nth term rule using CASIO classpad calculator

Example.

The 4th term of an arithmetic sequence is 17, and the 9th term is 32.

Find 'a' and 'd', then find the sequence rule.

Step 1: Find the simultaneous equations button. It's located in: Menu → Main → Keyboard (physical button) → Math 1

Step 2: Enter the two sequence rules with your values, solve for a and d.

$$\begin{cases} a + (4-1)d = 17 \\ a + (9-1)d = 32 \end{cases}\Bigg|_{a,d}$$

Step 3: Hit EXE to calculate $\{a = 8, d = 3\}$

Step 4: Sub 'a' and 'd' into the nth term rule $t_n = 8 + (n-1)3$

Finding the nth value using CASIO classpad calculator

The rule is (*from the example above*): $t_n = 8 + (n-1)3$. Find the 20th term

Step 1: Enter the nth term rule in: Menu → Sequence → Explicit
$a_nE = 8 + (n-1)3$ *(you need to use the 'n' from the menu up the top).*
Hit the EXE button or tick the rule.

Step 2: Press ♦ → Σdisplay → On

Step 3: +−/× Σan → X Y → Start = 1, End = 20 →

Step 3: The calcualtor will display a table (example below). n is the term position, a_nE is the term value, and Σa_nE is the sum of the series up to that point.

n	a_nE	Σa_nE
1	8	8
2	11	19
3	14	33
4	17	50

← The 2nd term is 11, the sum of the first 2 terms is 19.

← The 4th term is 17, the sum of the first 4 terms is 50.

With your teacher.

1. How many terms of the sequence: 10.5, 11, 11.5, 12 … must be added to obtain 159.

2. Soup cans are displayed in a supermarket arranged in a triangular stack. How many cans in total will be in the stack if it is 13 rows high?

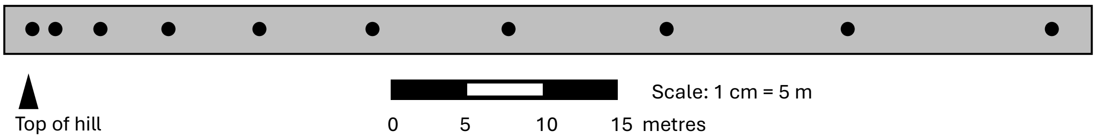
3. A cyclist training for a race event begins by setting herself a weekly distance target of 200km. Each week she aims to extend her training by riding an extra 50km.
   a. Write the first four terms of the sequence.

   b. If the event is ten weeks away find the distance that the cyclist will be riding in the tenth week.

   c. Find the total distance that the cyclist rides during the ten weeks of training.

With your teacher.

A painter's van rolled down a hill after being parked, crashing into a Gelato cart at the bottom of the hill. A leaky paint drum in the back of the van dripped once every second, leaving a trail of paint spots on the road as the van rolled away. The diagram below shows the start of the pattern of drips.



a. What does the trail of dots show?

b. Use the scale the diagram to find the distance travelled during each second. Complete the table below.

Second No.	0	1	2	3	4	5	6	7	8	9
Distance travelled	0									

c. Show that the distance travelled each second is in the form of an arithmetic sequence and find the sequence rule.

d. Find the speed of the van when it caused the accident given that police estimate that the accident occurred after 16 seconds.

e. Use a formula to find the total distance travelled from the top of the hill to the accident site.

Try it yourself (ARITHMETIC SERIES) *Ans pg78*

1. Use a formula to evaluate:
   a. 3 + 6 + 9 + 12 + 15 + 18 + 21
   b. 13 + 17 + 21 + 25 + 29 + 33
   c. -16 + -11 + -6 + -1 + 4 + 9
   d. 23.4 + 23.8 + 24.2 + 24.6 + 25.0 + 25.4 + 25.8 + 26.2 + 26.6
   e. $2\frac{1}{4} + 2 + 1\frac{3}{4} + 1\frac{1}{2} + 1\frac{1}{4} + 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4}$
   f. -115 + -122 + -129 + -136 + -143 + -150

2. Find the sum of:
   a. 13, 18, 23, 28, ….. to 10 terms
   b. -7, -4, -1, 2, ….. to 18 terms
   c. 10, 8, 6, 4, ….. to 15 terms
   d. 32, 42, 52, 62, …… to 16 terms
   e. 5.0, 5.5, 6.0, 6.5, ……to 20 terms
   f. -4, -8, -12, -16, …… to 27 terms

3. The first term of an arithmetic sequence is 17. The second term is 20. Find the sum of the first ten terms.

4. The first term of an arithmetic sequence is 7. The second term is 1. Find the sum of the first eight terms.

5. For the below sequences, how many terms must be added to obtain the total stated in the brackets? (hint: use the formula, then solve for n)
   a. 12, 14, 16, ….. (432).
   b. 11, 7, 3, ….. (-132).
   c. 3.5, 3.9, 4.3, ….. (378).
   d. 102, 94, 86, ….. (702).
   e. $1\frac{3}{4}$, $1\frac{1}{2}$, $1\frac{1}{4}$, ….. (0).
   f. -11.5, -6.5, -1.5, ….. (352.5).

6. A company offers workers an annual salary of $62,000 with an annual pay rise of $2,500 for each year of continuous employment. Use formulae to find the salary of an employee during year 5 and the total earned over 5 years of continuous employment.

Try it yourself (ARITHMETIC SERIES cont) *Ans pg78*

7. Tennis Club membership fee has increased by $10 each year since the club began. In the year the club began the fee was $52. Find:
   a. A rule for finding the membership fee for the n^{th} year.
   b. Find the membership fee for the eighth year.
   c. The total paid by a member who joined in the opening year and held membership for 8 years.

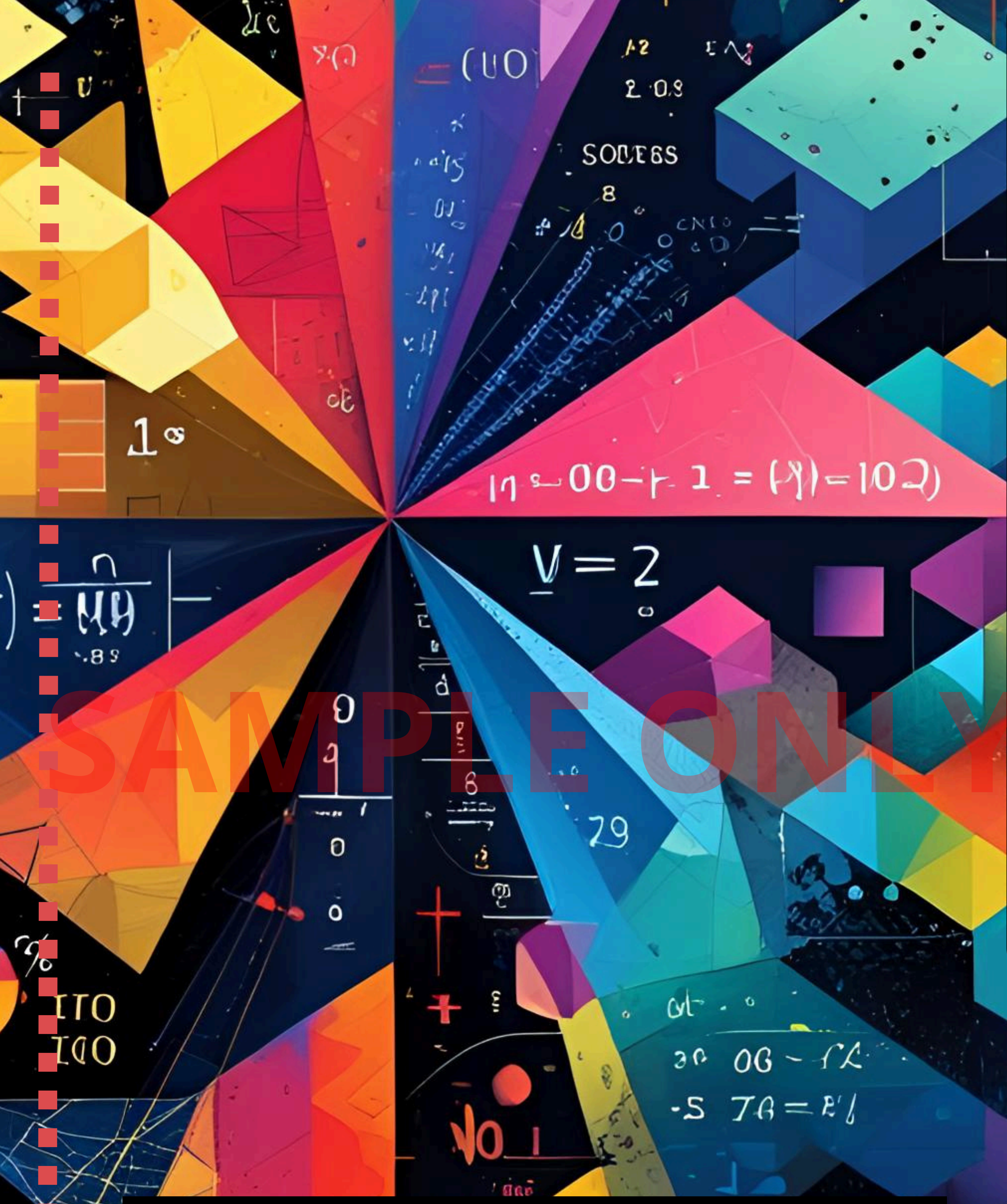
8. Some children are swimming from a beach to a dinghy where their father is fishing, and then back to the shore again. At first the dinghy is 65m from the beach but each time the children swim out the dinghy has drifted a further 20m from the beach. The children make 8 return trips to the dinghy. How far do the children swim?

9. Archie's nan gave him $20 for his first birthday in a piggy bank. Every subsequent birthday Archie's nan continued the tradition but raised the yearly gift by $5. Archie put his nan's money in the piggy bank every year.
   a. How much did Archie's nan give for Archie's 21^{st} birthday?
   b. How much was in Archie's piggy-bank after his 21^{st} birthday?

10. A small printing business produces custom posters. On the first day they print 240 posters, and each day they print 15 more than the day before.
    a. Find how many posters are printed on day 12.
    b. Find the total number of posters printed in the first 12 days.

11. A teacher gives her class a sequence of weekly homework tasks. In week 1 each task takes 25 minutes, and the time required increases by 5 minutes each week.
    a. How long will the 10th task take?
    b. What is the total amount of time a student spends on homework over the first 10 weeks?

12. A water tank is drained gradually. At the start there are 1,000 litres of water. Each hour, 40 litres are drained off.
    a. Write a rule for the volume of water after each hour.
    b. Find how much water remains after 12 hours.
    c. Find the total amount of water drained in the first 12 hours.



GM3 Sequences

2026 version

Jess Bertram

BERTRAM EDUCATION