2026

GENERAL MATHEMATICS

Level 3

FINANCE

General Mathematics: Level 3

GM3 – FINANCE

By Jess Bertram

With sincere thanks to John Short and Rick Smith.

ICON:	MEANING:
	Worked example
	Complete with your teacher
	Try it yourself
	CAS Calculator can be used
	Tips / shortcuts

For ordering, feedback and requests please visit:

BertramEducation.com

GENERAL MATHEMATICS (MTG315123)

INFORMATION SHEET

Finance

Simple interest: $I = PRT$

Compound interest: $FV = PV(1+i)^n$

Straight line depreciation: $V = -Dn + C$

Reducing value depreciation: $V = C(1-i)^n$

Effective interest: $E = (1+i)^n - 1$

Annuities in advance/Sinking funds: $F = \dfrac{R(1+i)[(1+i)^n - 1]}{i}$ OR $t_{n+1} = r(t_n + d) \quad t_0 = 0$

Present value of an annuity: $PV = \dfrac{FV}{(1+i)^n}$

Annuities in arrears/Reducible balance: $P = \dfrac{R[1-(1+i)^{-n}]}{i}$ OR $t_{n+1} = rt_n - d \quad t_0 = a$

Perpetuities: $P = \dfrac{R}{i}$

This information was correct when these books went to print (end of 2025). Please check no changes have been made during 2026 prior to the exam.

GENERAL MATHEMATICS - LEVEL 3

GM3 FINANCE

BY JESS BERTRAM

INDEX PAGE

Part 1 COMPOUND INTEREST

REVISION OF SIMPLE INTEREST

Before we dive into compound interest, we are going to review the basics of simple interest. Simple interest is the interest earned or paid only on the original principal. Unlike compound interest, it does not accumulate on past interest.

Simple interest formula:

$$I = PRT$$

Where:

(I) = Amount of interest earned or paid

(P) = Principal (initial amount invested or borrowed)

(R) = Interest rate per annum (as a fraction or decimal)

(T) = Time (in years)

Worked Examples.

1. You invest $5,000 at 6% simple interest for 3 years. Calculate the interest earned and the total balance of the account after 3 years.

   $I = \$5,000 \times 0.06 \times 3$

   $I = \$900$

   There was $900 of interest earned, making a total balance of $5,900.

Complete With Your Teacher:

1. Calculate the interest on a $2,000 investment at 4% simple interest for 2 years.

With your teacher.

2. $750 is invested for 5 years at 8% p.a. simple interest.

   a. How much interest does it earn?

   b. What is the total amount returned to the investor?

3. A loan of $ 2800 is borrowed at an interest rate of 18% p.a. It is to be repaid in 31 days. Find the total repaid.

> The simple interest formula (I = PRT) can also be used to find the principal (P), interest rate (R), and time (T).

Worked Examples.

Craig invests $7,000 at 5% p.a. simple interest. If he earns $600 in interest how long was it invested for?

$$\begin{aligned} I &= PRT \\ \$600 &= \$7{,}000 \times 0.05 \times T \\ T &= \$600 \div (\$7{,}000 \times 0.05) \\ T &= 1.714 \text{ years or approx. 1 year and 9 months} \end{aligned}$$

With your teacher.

1. For how long (in years and months) must $1250 be borrowed at 7 % pa flat before the interest exceeds $ 400?

2. For how long (in months) must $ 4500 be invested at 4.2 % pa. flat to grow to $6,000?

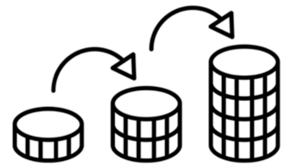
3. Rodney West repays a $5000 loan over 18 months. Including interest, he repays a total of $ 6200 in all. Find the flat rate of interest over the course of the loan.

4. Freya Behrens takes out a ‘pay day’ loan to finance her holiday. (A pay day loan is a short term loan offered to applicants with secure employment over a short term. Such loans often involve a high interest rate.) She must repay a total of $ 820 on her $ 650 loan within 2 months. Find the interest rate involved.

Try it yourself (SIMPLE INTEREST): *Answers page 76*

1. Find the interest amount earned on the following:
    a. $3,500 invested for 5 years at 3%
    b. You invest $7,200 at 4.2% for 4 years.
2. Find the original amount (principal) invested in the following:
    a. You pay $268 in interest after 2 years. Interest rate was 8%.
    b. A total of $660 interest is paid over 4 years at 6%.
3. Find the annual interest rate for the following:
    a. A $9,000 earns $675 interest in 2.5 years.
    b. You earn $960 interest on a $4,000 investment over 4 years.
4. Find the amount of time (in years) for the following:
    a. $600 interest is earned from a $5,000 deposit at 6% interest.
    b. You receive $240 interest from a $2,000 investment at 4% p.a.
5. How much interest would be earned by an investment of $15,000 over a quarter of a year at a flat rate of 7.3% pa.
6. Shaun has a savings account which pays interest of 5.2% pa based on the daily balance. At the end of July, Shaun's balance was $5,942.20.
    a. How much interest will be awarded for the month of August if Shaun has not used his bank account?
    b. How much will then be in his account?
7. For how long (in years and months) must $1,460 be invested at 7.5 % flat before the interest has exceeded $ 200?
8. For how long (in years and months) must $1,750 be invested at 8 % flat before the interest has exceeded $100?
9. For how long must $ 6000 be invested at 8 % pa. flat to grow to $8,000?
10. Wendy is repaying a loan of $ 600 over 2 years. Including interest she repays a total of $680. Find the flat rate of interest over the loan.
11. Simone is repaying a loan of $1,200 over 2 years. Including interest she repays a total of $1,464. Find the flat rate of interest over the loan.
12. Graeme took out a loan of $800. Over a period of 3 months he repaid the lender a total of $872. Find the flat rate of interest on the loan.

INTRODUCTION TO COMPOUNDING INTEREST



Compound interest means that the interest is added to the principal after each period. The principal, having been increased will earn more interest during the second interest bearing period than during the first. This second interest instalment is also added to the principal, increasing it again - and the cycle continues.

Worked example.

Alice deposits $1,000 in an account paying 6% pa compound interest calculated annually. How much will be in the account at the end of 3 years?

The problem can be solved by using the simple interest formula:

Interest for 1st year $I = Prt$

$I = 1000 \times 0.06 \times 1$

$I = \$60$

Amount in account after first year is $1,000 + 60 = $1,060

Interest for 2nd year $I = Prt$

$I = 1060 \times 0.06 \times 1$

$I = \$63.60$

Amount in account after second year is $1,060 + $63.60 = $1,123.60

Interest for 3rd year $I = Prt$

$I = 1123.60 \times 0.06 \times 1$

$I = \$67.41$

Amount in account after third year is $1,123.60 + 67.41 = $1,191.02

There is a compound interest formula that is useful to calculate the balance at any time. Before we jump into the formula, it's important to understand how balances change step by step using recurrence relations.

What is a Recurrence Relation?

A recurrence relation is an equation that defines each new term of a sequence in terms of the previous term. In finance, it models how the balance of a loan or investment changes from one compounding period to the next.

You may also see recurrence relations referred to as Difference Equations

Recurrence relation formula

$$t_{n+1} = rt_n + d$$

Where:

(r): Rate of growth $(1 + i)$ or decay $(i - 1)$

(i): Interest rate per period

(t_n): Balance after (n) compounding periods

(d): Deposit amount each period (+ for deposit, - for withdrawal/repayment)

Worked examples.

1. You deposit $1,000 into a savings account that earns 6% annual interest, compounded annually. Write the recurrence relation (or difference equation) and calculate the balance after 3 years.

The recurrence relation is:

$$t_{n+1} = 1.06 \times t_n \qquad t_0 = \$1,000$$

Step-by-step calculation:

Year 1: $t_1 = (1.06) \times 1{,}000 = 1{,}060$

Year 2: $t_2 = (1.06) \times 1{,}060 \quad = 1{,}123.60$

Year 3: $t_3 = (1.06) \times 1{,}123.60 = \mathbf{1,191.02}$ ← The balance after 3 years is $1,191.02

2. A loan of $10,000 has 12% annual interest, with $500 monthly repayments. Write the difference equation and calculate the balance for the first 3 years.

The recurrence relation is:

$$t_{n+1} = \left(1 + \frac{0.12}{12}\right) \times t_n - 500, \qquad t_0 = \$10,000$$

Step-by-step calculation:

Month 1: $t_1 = (1.01) \times 10{,}000 - 500 = 9{,}600.00$

Month 2: $t_2 = (1.01) \times 9{,}600 - 500 = 9{,}196.00$

Month 3: $t_3 = (1.01) \times 9{,}196 - 500 = 8{,}787.96$

Starting balance	Interest added	Repayment	Closing balance
$10,000	$100.00	$500	$9,600.00
$9,600	$96.00	$500	$9,196.00
$9,196	$91.96	$500	$8,787.96

Table format

Key Takeaways of recurrence relations:

- Recurrence relations let you calculate balances period by period.
- They help you understand how compound interest accumulates over time or how loans are gradually paid off.
- This approach is the foundation for the compound interest formula, which we will learn next.

Complete with your teacher:

1. An investment of $2,500 earns 5% annually. Write the recurrence relation and calculate the balance after 5 years.

2. A loan of $5,000 has 1% monthly interest and $200 monthly repayments. Write the recurrence relation and calculate the balance after 3 months.

3. Complete a table showing the balance of a $1,200 investment earning 8% annually for 4 years.

Try it yourself (RECURRENCE RELATIONS): *Ans pg76*

1. A deposit of $1,500 earns 7% annual interest, compounded annually.
    a. Write the recurrence relation.
    b. Calculate the interest for each of the first 4 years.
    c. How much interest was earned in total after 4 years?

2. A loan of $8,000 has a monthly interest rate of 0.9% with repayments of $300 each month.
    a. Write the recurrence relation.
    b. Calculate the balance after 1, 2, and 3 months.
    c. Explain how the repayments affect the remaining balance over time.

3. You invest $600 at 10% annual interest.
    a. Write the recurrence relation.
    b. Calculate the balance for each of the first 3 years.

4. $3,000 is deposited into an account earning 4% interest annually. At the end of each year, an additional $500 is deposited.
    a. Write a recurrence relation that includes the yearly deposit.
    b. Calculate the balance for the first 4 years.

5. Account A: $2,000 invested at 6% annual interest. Account B: $2,000 invested at 6% annual interest with an additional $200 added at the end of each year.
    a. Write the recurrence relations for both accounts.
    b. Create a table for each account (like the one in worked example 2), showing the balances and interest added each year over the course of 5 years.
    c. How much of a difference did the additional $200 make over the 5 years?

6. You take out a loan of $10,000 with an interest rate of 20% and monthly repayments of $800. Write the difference equation and calculate the balances for the first 6 months.

THE COMPOUND INTEREST FORMULA

Now that you understand recurrence relations and how balances can change period by period, we will explore a faster way to calculate the same results: the compound interest formula.

Compound interest formula

$$FV = PV(1 + i)^n$$

Where:

FV = Future Value

PV = Present Value

i = Interest rate per period (as a decimal or fraction)

n = Number of interest bearing periods

In the previous section, we used recurrence relations to calculate how money grows or how a loan balance changes over time. Now, let's see how we can use a formula to get the same result more quickly. We'll compare both methods side by side.

Worked example: Comparing methods

You invest $2,000 at 5% annual interest, compounded annually for 5 years.

Recurrence relations:	Compound formula:
Year 1: 1.05 x $2,000 = $2,100 Year 2: 1.05 x $2,100 = $2,205 Year 3: 1.05 x $2,205 = $2,315. 25 Year 4: 1.05 x $2,315.25 = $2,431.01 Year 5: 1.05 x $2,431.01 = $2,552.56	$FV = PV(1 + i)^n$ $FV = \$2{,}000(1 + 0.05)^5$ $FV = \$2{,}552.56$

Comparison:

- Both methods give the same result.
- Recurrence relations calculate balances period by period, which is helpful for understanding how compound interest works.
- The formula is a shortcut for quickly finding the future value over many periods, imagine doing recurrence relations for 30 years!

Worked Examples.

1. You deposit $1,000 at 6% annual interest, compounded annually for 3 years. Find the future value.

$$FV = \$1{,}000(1 + 0.06)^3$$
$$FV = \$1{,}191.02$$

2. An investment of $5,000 earns 8% annual interest, compounded quarterly for 2 years. Find the value after 2 years.

$$FV = \$5{,}000\left(1 + \frac{0.08}{4}\right)^{2\times4}$$
$$FV = \$5{,}000(1 + 0.02)^8$$
$$FV = \$5{,}858.30$$

With your teacher:

1. An investment of $2,000 earns 5% annual interest, compounded annually for 6 years. Find the future value.

2. A loan of $10,000 has 7% annual interest, compounded monthly. What is the amount owed after 5 years if no payments are made?

Try it yourself (COMPOUND INTEREST FORMULA). *Ans pg76*

1. Explain why using the formula is more practical for long-term investments.
2. Use the compound interest formula to find the future value of the following:
   a. $8,500 invested at 3%p.a. compounding monthly for 3 years.
   b. 2.5%p.a. interest rate, compounding weekly for 2 years. $9,000 invested.
   c. Quarterly compounding, 10% annual interest, 5 years, $7,560 invested.
   d. $200 invested at 6%p.a. compounding fortnightly for 10 years
   e. $500 invested for 60 years. Interest rate of 4% p.a. compounding yearly.
3. A deposit of $1,200 earns 10% annual interest, compounded quarterly for 3 years. Find the future value.
4. An investment of $1,500 earns 6% annual compounding interest for 3 years. Use both recurrence relations and the compounding formula to find the future value.
5. $3,000 is invested at 8% annual interest, compounded annually for 5 years. Compare results using both methods.
6. A loan of $10,000 has 7% monthly compounding interest. Calculate the amount owed after 2 years with both methods.
7. Calculate the future value for each of the below after 5 years and decide which is better. Explain why the values differ despite having the same interest rate.

   Investment A: $1,500 at 8% annual interest, compounded annually.
   Investment B: $1,500 at 8% annual interest, compounded quarterly.
   Investment C: $1,500 at 8% annual interest, compounded monthly.
   Investment D: $1,500 at 8% annual interest, compounded daily.
8. Mia wants to save for a big overseas holiday. She deposits $4,000 into a high-interest savings account that offers 4.5% annual interest, compounded annually. Mia plans to leave the money untouched for 5 years. How much money will Mia have saved by the time she's ready to book her trip?
9. Alex invests $7,500 for 4 years. Interest is added half-yearly. For the first two years the rate per annum is 8 % and for the remaining 2 years, the rate per annum is 10 %. How much does he have in the end?

CHANGING VARIABLES

In this section, we'll explore how three different factors affect the future value of loans and investments:

- interest rates
- number of periods
- compounding frequency

By changing these variables, you can see how your money grows or how much you'll owe. Understanding these impacts helps you make smarter financial decisions.

Changing the Interest Rate

Worked example: $1,000 invested for 30 years:

At 5%	At 10%	At 15%
$FV = \$1{,}000(1 + 0.05)^{30}$ $FV = \$4{,}321.94$	$FV = \$1{,}000(1 + 0.1)^{30}$ $FV = \$17{,}449.40$	$FV = \$1{,}000(1 + 0.15)^{30}$ $FV = \$66{,}211.77$

With your teacher: $5,000 is invested for 10 years.

a. Calculate the future value at the interest rates of 4%, 6%, and 8% (compounded annually).

4%	6%	8%

b. Discuss how increasing the interest rate affects the total amount earned.

Changing the Number of Periods (Time)

Worked example: $1,000 is invested at 6% compounding annually:

5 years	10 years	20 years
$FV = \$1{,}000(1 + 0.06)^5$ $FV = \$1{,}338.23$	$FV = \$1{,}000(1 + 0.06)^{10}$ $FV = \$1{,}790.85$	$FV = \$1{,}000(1 + 0.06)^{20}$ $FV = \$3{,}207.14$

With your teacher: A loan of $10,000 is taken out at 7% annual interest.

a. Calculate the total amount owed if the loan is left unpaid for 3 years, 6 years, and 9 years.

3 years	6 years	9 years

b. Discuss the effect of time on the growth of the loan balance.

Changing Compounding Frequency

Compounding can occur annually, six-monthly, quarterly, monthly, weekly, or even daily. More frequent compounding increases the future value because interest is added more often.

Worked example: $1,000 invested at 12% p.a. for 30 years:

Annual compound	Quarterly	Monthly
$FV = \$1{,}000(1 + 0.12)^{30}$ $FV = \$29{,}959.92$	$FV = \$1{,}000(1 + \frac{0.12}{4})^{30\times4}$ $FV = \$34{,}710.99$	$FV = \$1{,}000(1 + \frac{0.12}{12})^{30\times12}$ $FV = \$35{,}949.64$

With your teacher: $2,000 is invested at 10% annual interest for 25 years.

a. Calculate the future value if interest is compounded annually, quarterly, and monthly.

Annual compound	Quarterly	Monthly

b. Discuss how increasing the compounding frequency changes the final amount.

Try it yourself (CHANGING VARIABLES): *Ans pg77*

1. Calculate the future value of $2,000 at 4% annual compound for 5, 10, and 15 years.
2. Compare the future value of $5,000 invested for 10 years at 2%, 6%, and 8% interest, all compounding monthly.
3. Calculate the future value of $3,000 invested at 12% p.a. interest for 2 years with: annual compounding, monthly compounding, daily compounding (365 days)
4. A loan of $20,000 at 10% p.a. compounding weekly is unpaid for 3, 6, and 9 years. Calculate the amount owed each time.
5. $1,000 is invested for 5 years. Calculate the future value at 3%, 5%, and 7% annual interest, compounded fortnightly. Compare the results.
6. A savings account holds $10,000 for 15 years. Find the future value at 4%, 6%, and 9% annual interest rates, compounding monthly.
7. A $20,000 investment grows for 20 years. Calculate the future value at 5%, 10%, and 15% annual interest, compounding annually.

8. A loan of $2,500 is left unpaid at 8% p.a. compounding weekly. Find the amount owed after 2, 4, and 6 years. Explain how time impacts the debt.
9. An investment of $15,000 grows at 6% p.a. compounding quarterly. Calculate the future value after 10, 20, and 30 years.
10. $12,000 is deposited into an account earning 10% interest p.a. for 8 years. Calculate the future value with annual, quarterly, and monthly compounding.
11. Mr Britcliffe has been given a million dollars to invest for his retirement (in 10 years). There are three superannuation funds that he is trying to choose between that all offer an annual rate of 8%. However they have different compounding periods, one is monthly, one is fortnightly, and one is weekly. Calculate the future value of the investment for each and state which superfund would be best.

SOLVING FOR VARIABLES IN THE COMPOUND INTEREST FORMULA

We have learned how to calculate the future value (FV) of investments and loans using the compound interest formula. In this section, we will go further and solve different types of problems where we might need to find:

- The Present Value (PV)
- The Interest Rate (i)
- The Number of Compounding Periods (n)

All of these can be solved using rearranged formulas. Solving for interest rate or number of periods can get complicated – it is recommended to use your CAS calculator.

CAS Calculator – finance solver

Menu –→ Financial → compound interest

N = Number of payment (instalment) periods

I% = Annual interest rate (as a %)

PV = Present value

PMT = Amount paid each period

FV = Future value

P/Y = Payments (instalments) per year

C/Y = Compounding periods per year

Notes: *In format – ensure payments are set to 'end of period'.

*Always make either PV or FV negative - it doesn't matter which is which

Worked example:

Moya wants to withdraw $15,000 in 7 years from an account that earns 3.5% p.a. compounding monthly. How much should she invest now to make that happen?

Enter the values on the right into the finance solver. Then select PV and press SOLVE. Remember you will get a negative value for PV if you put a positive value in FV.

PV = $11,744.76

N = 7
I% = 3.5
PV = SOLVE
PMT = 0
FV = 15000
P/Y = 12
C/Y = 12
END

Finding Present Value (PV) – rearranged formula

$$PV = \frac{FV}{(1+i)^n}$$

Worked example 1:

You want $10,000 in 5 years and can earn 6% p.a. compounding annually. How much should you invest now?

Using rearranged formula:

$$PV = \frac{\$10{,}000}{(1+0.06)^5}$$

$$PV = \$7{,}472.58$$

Need to invest $7,472.58 today.

Using CAS calculator:

N = 5

I% = 6

PV = SOLVE

PMT = 0

FV = 10000

P/Y & C/Y = 1

END

Worked example 2:

Irene invests some money and is told it will be worth $25,000 in 10 years. She is going to earn an interest rate of 4.5% p.a., compounding weekly. How much did she invest?

Using rearranged formula:

$$PV = \frac{\$25{,}000}{(1+\frac{0.045}{52})^{520}}$$

$$PV = \$15{,}943.81$$

She invested $15,943.81

Using CAS calculator:

N = 10 x 52

I% = 4.5

PV = SOLVE

PMT = 0

FV = 25000

P/Y & C/Y = 52

END

With your teacher:

How much needs to be invested now to have $12,000 in 10 years at 6% p.a. compounding fortnightly?

Worked examples:

1. You invest $2,000 and after 4 years have $2,400. Find the interest rate if the investment compounds annually.

Using CAS calculator:

N = 4

I% = SOLVE → **4.66%**

PV = 2000

PMT = 0

FV = -2400

P/Y = 1

C/Y = 1

END

2. You invest $3,000 at 5%p.a. compounding annually. How many years until it doubles?

Using CAS calculator:

N = SOLVE → **14.2 years**

I% = 5%

PV = 3000

PMT = 0

FV = -6000

P/Y = 1

C/Y = 1

END

With your teacher:

1. An investment of $2,500 grows to $3,000 in 3 years. Find the annual interest rate if the investment compounds monthly.

2. How many years will it take for $4,000 to grow to $8,000 at 7%p.a. compounding monthly?

3. How long will it take for $10,000 to reach $15,000 at 4%p.a. compounding fortnightly?

Try it yourself (SOLVING FOR VARIABLES - COMPOUNDING): *Ans pg77*

1. Find the present value of the following investments:
    a. $2,000 in 5 years with 4% interest compounding monthly.
    b. $10,000 received in 6 years with 6%p.a. compounding fortnightly.
    c. 3% p.a. compounding daily (365 days) for 10 years, $52,500 final amount.
2. Sarah wants $3,500 in her account in 10 years. If she makes one deposit today and leaves it untouched at 5% p.a. compounding yearly, how much does she need now?
3. A company needs $50,000 in 10 years for equipment replacement. If the interest rate is 7%p.a. compounding monthly, how much should they set aside today?
4. Tom wants to buy a house in 12 years and estimates it will cost $400,000. How much should he invest now at 5% annual interest to reach this goal?
5. Calculate the annual interest rate for the following: (round to 2 decimal places)
    a. $2,000 becomes $2,500 after 4 years. Compounds weekly.
    b. $1,000 grows to $1,100 in 2 years. Compounds fortnightly.
    c. A savings account that compounds monthly triples in value over 15 years.
6. Tim wants to know what interest rate he would need to get if he wants his $50,000 house deposit to grow to a $120,000 in the next 5 years. Interest compounds monthly and he doesn't plan on making any additions to the $50k.
7. Calculate how many years the following amounts were invested:
    a. $4,000 reaches $10,000 after being invested at 8%p.a. compounding weekly.
    b. Retirement fund grows from $50,000 to $150,000 at 7% compounding yearly.
    c. $3,000 doubles when invested at 12% p.a. compounding daily (365 days).
8. Jane is trying to figure out when she can retire. She has a lump sum of $1,000,000 and estimates she will need $1,200,000 to retire comfortably. She can get a deal that offers 8% p.a. compounding monthly. How long until she can retire?

Try it yourself (MIXED COMPOUND INTEREST PROBLEMS) *Ans pg77*

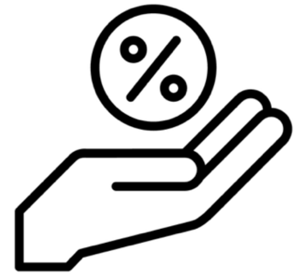
1. $ 1500 is invested at 6 % pa interest compounding annually. Find the amount in the account after 6 years.
2. Find the total in an account if:
   a. $3,000 is invested at 8% pa interest compounding quarterly for 5 years.
   b. $1,250 is invested at 9% pa interest compounding monthly for 7 years.
   c. $3,428 is invested at 4% pa interest compounding quarterly for 6 years.
   d. $2,500 is invested at 5.5% pa interest compounding monthly for 5 years.
   e. $1,600 is invested at 6.25% pa compounding twice per year for 4.5 years.
   f. $7,250 is invested at 5.8% pa interest compounding quarterly for 5 ½ years.
3. $4,500 is invested at 6% pa compounding quarterly.
   a. Find the amount in the account after 6 years.
   b. Find the amount of interest earned.
4. $5,250 is invested at 5.5 % pa compounding quarterly.
   a. Find the amount in the account after 4 years.
   b. Find the amount of interest earned.
5. Alex Jones invests $7,500 for 4 years. Interest is added half-yearly. For the first two years the rate per annum is 8% and for the remaining 2 years, the rate per annum is 10%. How much does he have in the end?
6. Jacqui wishes to invest $4,000 for 6 years. Which of the following provides the best investment opportunities? (In each case, give the final amount that would be returned to the investor):
   a. simple interest at 7% per year
   b. interest compounded annually at 7%
   c. interest compounded half yearly at a rate/annum of 7%
   d. interest compounded monthly at a rate/annum of 7%.
7. A man wishes to invest $5,000 for 10 years. Find his best investment alternative given the following choices:
   a. simple interest at 8% per annum;
   b. compound interest at 7% pa adjusted annually;
   c. compound interest adjusted monthly at a rate of 6% pa.

Try it yourself (MIXED COMPOUND INTEREST PROBLEMS) *Ans pg77*

8. In your own words explain why an investment which offers 6% pa interest compounded monthly will be better than one which offers 6% pa interest compounded annually.
9. Draw a graph which shows the growth of a $1,000 investment at 9% pa interest compounding monthly over 20 years. Explain why the graph has its shape.
10. Calculate the principal required to achieve $20,000 in 10 years if interest is added quarterly and the annual rate is 6%.
11. Calculate the principal required to achieve $5,000 in 6 years if interest is added monthly and the annual rate is 6.75%.
12. Jill needs $7,000 for a trip in 3 year's time. Her credit union offers a rate per annum of 5% which will be compounded quarterly. How much should she deposit (to the nearest dollar)?
13. Now use your calculator's financial mode to recalculate some of the problems in this set as directed by your teacher.

14. Use your calculator's financial mode to find:
    a. The interest rate required for $1,000 to increase to $1,500 in 5 years with interest added annually.
    b. The interest rate required for $2,500 to increase to $3,000 in 5 years with interest added quarterly.
    c. The interest rate required for $5,000 to increase to $8,000 in 10 years with interest added monthly.
    d. The time required for $1,000 to increase to $1,500 at 5% compound interest pa added annually.
    e. The time required for $3,500 to increase to $4,000 at 8% compound interest pa added monthly.
    f. The time required for $6,000 to increase to $7,000 at 6% compound interest pa added quarterly.

EFFECTIVE ANNUAL RATE OF INTEREST



Banks often advertise a nominal annual rate. This is the advertised annual rate, which does not take into account how often interest is compounded.

Because interest can be compounded more than once a year (e.g., monthly or quarterly), the actual interest earned or paid is different. This is called the Effective Annual Rate (EAR).

Definition: ***The Effective Annual Rate (EAR) is the actual interest rate you earn (or pay) in a year, accounting for compounding periods.***

Why EAR Matters:

- Allows you to compare different investments or loans with varying compounding frequencies.
- Shows the true cost of borrowing or true return on investment over a year.
- Helps you make better financial decisions.

Effective Annual Rate Formula:

$$E = (1+i)^n - 1,$$

Where:

(i): Interest rate per compounding period (as a decimal or fraction)

(n): Number of compounding periods per year

Worked example – Compare the following bank accounts:

Bank A: 12% interest p.a., compounded annually

Bank B: 12% interest p.a., compounded monthly

Bank A	Bank B
$E = (1+0.12)^1 - 1$ $E = 0.12 \times 100 = 12\%$	$E = (1+\frac{0.12}{12})^{12} - 1$ $E = 0.1268 \times 100 = 12.68\%$

Bank B offers a slightly higher return because interest is compounded more often.

With your teacher:

1. A Westpac savings account offers 6% annual interest, compounded monthly. Calculate the Effective Annual Rate (EAR).

2. A CommBank loan has a nominal annual interest rate of 18% with daily compounding (365 days). Calculate the EAR.

3. Compare two term deposits, which has the higher effective interest rate?
   - Deposit A: 8% annual interest, compounded quarterly.
   - Deposit B: 8% annual interest, compounded monthly.

Try it yourself (EFFECTIVE ANNUAL RATE): *Ans pg77*

1. Calculate the effective annual interest for the following:
    a. 10% interest p.a., compounding monthly.
    b. 14% p.a. interest, compounding fortnightly
    c. Quarterly compounding rate of 7.6% p.a.
2. A bank offers 5% annual interest, compounded annually. What is the EAR?
3. A term deposit has a nominal interest rate of 9% compounded quarterly. Calculate the EAR.
4. A savings account earns 4% annual interest, compounded monthly. Find EAR.
5. A loan advertises 15% annual interest, compounded daily (365 days). Find EAR.
6. Compare the two investments; Which has the higher EAR?
    Investment A: 10% annual interest, compounded annually.
    Investment B: 10% annual interest, compounded quarterly.
7. A credit card charges 20% annual interest, compounded monthly. Find the EAR.
8. An investment fund earns 18% annual interest, compounded fortnightly. Calculate the EAR.
9. A bank account offers 7% annual interest, compounded daily. How much higher is the EAR than the nominal rate?
10. Compare these options and state which option give the higher EAR.
    Option A: 12% annual interest, compounded monthly.
    Option B: 12.2% annual interest, compounded annually.
11. A savings account has 14% annual interest, compounded monthly. A different account has 13.8% annual interest, compounded daily. Calculate both EARs and explain which is better.
12. Find correct to two decimal places, the effective interest rate per annum if the nominal rate per annum is:
    a. 5% pa and interest is compounded monthly
    b. 6.5% pa and interest is compounded quarterly.
    c. 17.5% pa and interest is compounded half yearly.
    d. 15% pa and interest is adjusted daily.
    e. 8% pa and interest is adjusted monthly

Try it yourself (EFFECTIVE ANNUAL RATE). *Ans pg77*

13. The nominal interest on Gail's savings account, which is adjusted monthly, is 9.5% pa. Renee claims that her bank gives just as much interest even though it is added to her account just once yearly. Given that both girls have the same amount of savings find the interest rate offered by Renee's bank.

14. Sally has $1,600 which she wishes to invest. She has the following options:

    8.75% pa. compound interest (nominal) added yearly

    8.5% pa. compound interest (nominal) added quarterly

    8.45% pa. compound interest (nominal) added monthly

    a. Sally believes that she will be able to invest the money for about 4 years. Find her best investment option by considering the amount of interest earned by each alternative over this period of time.
    b. Check that this is the best by considering the effective interest rate per year in each case.
    c. If Sally chooses the best of the alternatives how long would it take before her investment had doubled in value?

15. Maria has $2,500 to invest and considers the following options:

    8.6% pa. (nominal) compound interest added yearly

    8.5% pa. (nominal) compound interest added half-yearly

    8.4% pa. (nominal) compound interest added quarterly

    8.3% pa. (nominal) compound interest added monthly

    a. Maria believes that the money will be invested for 5 years. Find the best investment option.
    b. Check your findings by finding the annual effective interest rate in each case.
    c. Choosing the best alternative, how long would it take before Maria's investment had doubled?
    d. What nominal annual rate of half-yearly adjusted compound interest would Maria need if she wished her investment to double in exactly 8 years?

COMPARING INVESTMENT STRATEGIES

Now that we understand how compound interest works, it's important to know how to compare different investment options. Small changes in interest rates, time, and compounding frequency can significantly impact the final amount of money you earn or owe.

> **Why Compare Investments?**
>
> When choosing between two or more financial products (like savings accounts, term deposits, or loans), you need to determine which offers the **best return** or the **lowest cost**. Compound interest calculations and the effective annual rate (EAR) help you make these comparisons accurately.

Example 1 – Comparing two savings accounts for $10,000 invested over 5 years.

Account A: 8% annual interest, compounded annually

Account B: 7.8% annual interest, compounded monthly

Account A

$$FV = \$10{,}000(1 + 0.08)^5$$

$$FV = \$14{,}693.28$$

Account B

$$FV = \$10{,}000(1 + \frac{0.078}{12})^{5\times 12}$$

$$FV = \$14{,}751.18$$

Account A gives a slightly better return.

Example 2 – Choosing a Loan for $20,000 over 4 years.

- **Loan A:** 6% annual interest, compounded annually
- **Loan B:** 5.8% annual interest, compounded monthly

Loan A

$$FV = \$20{,}000(1 + 0.06)^4$$

$$FV = \$25{,}249.44$$

Loan B

$$FV = \$20{,}000(1 + \frac{0.058}{12})^{4\times 12}$$

$$FV = \$25{,}139.05$$

Loan B costs less overall despite compounding more frequently.

With your teacher:

Investment A: $10,000 at 7% annual interest, compounded annually for 10 years.

Investment B: $10,000 at 6.8% annual interest, compounded quarterly for 10 years.

Calculate the future value of both investments manually using the compound interest formula. Discuss which investment gives a better return and why. Then use a CAS calculator to verify your answers.

Try it yourself (COMPARING OPTIONS). *Ans pg77*

1. Compare two investments, which is better after 7 years?

    Investment A: $5,000 at 9% annual interest, compounded annually.

    Investment B: $5,000 at 8.8% annual interest, compounded monthly.

2. Compare two term deposits; Which has the higher future value after 10 years for $10,000?

    Deposit A: 6.5% annual interest, compounded quarterly.

    Deposit B: 6.7% annual interest, compounded annually.

3. A bank offers two loan options; Which loan will cost less over 5 years for $15,000?

    Loan A: 11% annual interest, compounded annually.

    Loan B: 10.9% annual interest, compounded monthly.

4. An investment advisor presents two options; Compare the future values for a $2,000 investment over 15 years.

    Option A: 12% interest compounded annually.

    Option B: 11.8% interest compounded quarterly.

GM3 Finance

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