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EDUCATION

2027 Sample pack

General Maths level 2

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2027 sample pack

Discover how the course is structured through explicit theory, worked examples, guided practice and independent practice questions.
This sample pack includes select pages from each of the 5 units in the General Maths level 2 course.

Please note: The 2027 editions have been substantially revised and updated. To ensure page numbers, questions and teacher resources align, it is recommended that all students within a class use the same edition.

2027 available courses.

- General mathematics (level 2)
- General mathematics (level 3)

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Student workbooks

Overview

- Comprehensive workbook set covering the entire school year.
- Structured around the gradual release model (I do, We do, You do)

WHAT IS AN ARITHMETIC SEQUENCE?

An arithmetic sequence is a pattern where the numbers change by the same amount each step. This amount can be:

Positive	(increasing sequence)	→ 4, 7, 10, 13, 16, …	(+3 each time)
Negative	(decreasing sequence)	→ 15, 10, 5, 0, -5 …	(–5 each time)
Zero	(constant sequence)	→ 8, 8, 8, 8, …	(no change, +0)

Arithmetic sequences add or subtract the SAME number each time.

Arithmetic sequences model situations where something increases or decreases by a fixed amount over time. These sequences can represent real life situations.

Saving $50 every week	→ 50, 100, 150, 200, …
Temperature drops 2°C each hour	→ 18, 16, 14, 12, …
A plant grows 3 cm each day	→ 5, 8, 11, 14, …

Theory – Concepts are introduced using clear, student-friendly language.

Worked example. From sequence → graph

Jasmin starts a new job and earns $200 in her first week. Each week, she earns $50 more than the previous week as she takes on extra responsibilities.

This sequence could be written as: 200, 250, 300, 350, 400, 450, …

Step 1: Make a table of term position (n) and term value (t_n) for the first 6 terms.

Term position (n)	Value (t_n)		Point @ (n, t_n)
1	200	→	(1, 200)
2	250	→	(2, 250)
3	300	→	(3, 300)
4	350	→	(4, 350)
5	400	→	(5, 400)
6	450	→	(6, 450)

Step 2: Plot the points. We only plot the points (discrete values). We do not join them with a continuous line because sequences only exist at set intervals.

I do – Worked examples introduce each concept and model required skills.

With your teacher.

1. Identify if each sequence is arithmetic. If it is, state the common difference (d).

		Arithmetic sequence?	Common difference (d)
a)	3, 6, 9, 12, …		
b)	10, 7, 4, 1, …		
c)	5, 10, 20, 40, …		
d)	5, 5, 5, 5, …		
e)	2, 5, 9, 14, …		
f)	100, 90, 80, 70, …		

2. Write the first 5 terms (t_1, t_2, t_3, t_4, t_5) of each arithmetic sequence.
   a) Start at 4, add 6 each time

We do – Guided practice questions for students to complete with their teacher.

Try it yourself (INTRO TO SEQUENCES)

1. State whether each of the following sequences are arithmetic. If they are, state the common difference (d), first term (t_1) and the third term (t_3)
   a) 9, 12, 15, 18, …
   b) 16, 8, 4, 2, …
   c) 5, 3, 1, –1, –3 …
   d) 2, 5, 9, 14, …
   e) –10, –5, 0, 5, …
   f) 7, 7, 7, 7, …
2. Fill in the missing terms.
   a) 2, __, __, 8, 10
   b) __, 14, 11, 8, __
   c) –3, __, 1, __, 5
   d) 50, 45, __, 35, __

You do – Independent practice questions to build confidence and skill

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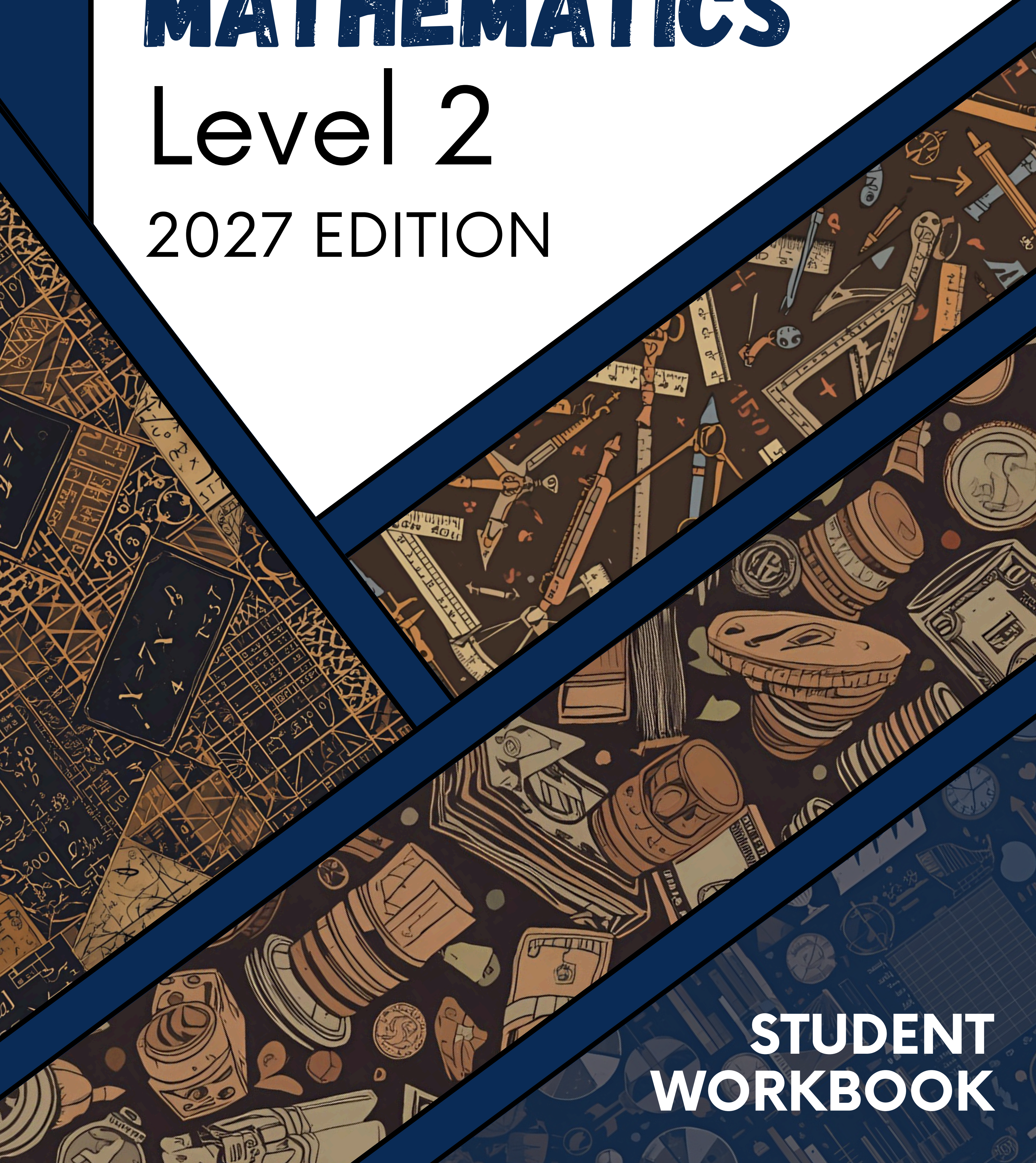
GENERAL MATHEMATICS

Level 2

2027 EDITION



STUDENT WORKBOOK

2027

GENERAL MATHEMATICS

Level 2

ALGEBRA

Part 1 ALGEBRA REVISION

DIRECTED NUMBERS

When working with positive and negative numbers (called directed numbers), signs can sometimes appear next to each other, such as +(–4) or – (–6). You can simplify these combinations using some basic rules:

> Two positives together give a positive result. +(+3) = +3
> Two negatives together give a positive result. – (–8) = +8
> A positive and a negative together give a negative result. +(–6) = –6

With your teacher.

Simplify the following:

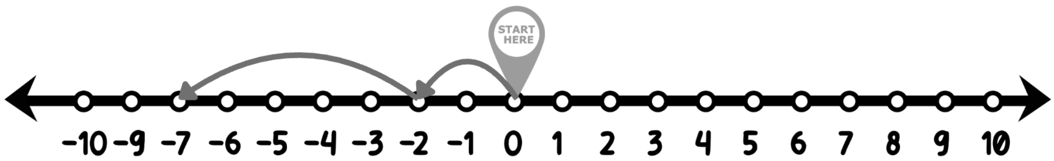
a) – (+38) = b) + –21 = c) – (–17) = d) – – 9 =

> **Adding and subtracting directed numbers.**
> When adding or subtracting you might like to imagine a number line.
> Positive moves to the right →, negative moves to the left ←.

Worked example.

Simplify: – 2 – 5

Start at 0. Move 2 to the left (-2) and then 5 more to the left (-5). The answer is -7.

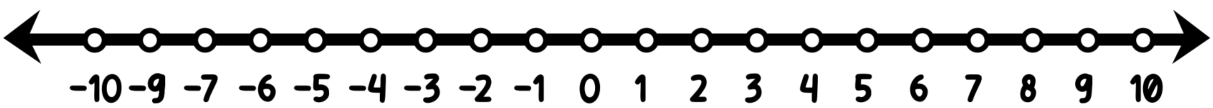


With your teacher.

Use the number line below to simplify the following:

a) $-4 - 1 =$ b) $-4 + 2 =$ c) $3 - 2 =$ d) $+9 - 11 =$

e) $4 - 8 =$ f) $11 + (-3) =$ g) $-2 - (5) =$ h) $10 - +18 =$



Multiplying and dividing directed numbers.

If the signs are the same the result will be positive,
if the signs are different the result will be negative.

Worked examples.

a) $\mathbf{-3 \times 2}$

$3 \times 2 = 6$

Different signs

$= -6$

b) $\mathbf{25 \div -5}$

$25 \div 5 = 5$

Different signs

$= -5$

c) $\mathbf{3 \times (1 + -4)}$

$3 \times (-3)$

$3 \times 3 = 9$

Different signs

$= -9$

Don't forget **BODMAS (Brackets, Other, Division and Multiplication, Addition and Subtraction)** when doing longer questions involving multiple operations.

Worked examples.

a) $\mathbf{3 \times (1 + -4)}$

$3 \times (-3)$

$3 \times 3 = 9$

Different signs

$= -9$

b) $\mathbf{+4 - (-3 + (-12 \div 2(-2)))}$

$+4 - (-3 + (-12 \div -4)$

$+4 - (-3 + 3)$

$+4 - 0$

$= 4$

With your teacher.

a) -7×6 b) -2×-5 c) $15 \div -3$ d) $-18 \div -3$

e) $4 \times 3 + -5$ f) $8 \times (2 + -6)$ g) $5 \times -3 + (-4 \times -2)$

Try it yourself (DIRECTED NUMBERS).

1. Simplify the following directed numbers.

a) $+(-5)$ b) $+(+13)$ c) $-(-4)$ d) $-+18$

e) $++7$ f) $-(+9)$ g) $-(-7)$ h) $+(-12)$

i) $+-21$ j) $-+14$ k) $-(-11)$ l) $-(+6)$

m) $+(+2)$ n) $-+16$ o) $+-17$ p) $+(-14)$

q) $+(+19)$ r) $-+9$ s) $-+11$ t) $+-10$

2. Calculate the following addition and subtraction questions

a) $+12-(-6)$ b) $-11-9$ c) $-5-(-8)$

d) $+3-10$ e) $+4+(-6)$ f) $-9+11$

g) $-8+(-7)$ h) $+13-(-2)$ i) $-6-(-4)$

j) $-12+3$ k) $+14-7$ l) $-3-15$

m) $+2+(-5)$ n) $-1-(-9)$ o) $+5-3+(-4)$

p) $-8+(-2)-5$ q) $+6-(-3)+(-7)$ r) $-10+4-(-6)+2$

s) $+9-12+(-5)$ t) $-(+4)-(-3)+(-2)$ u) $+[(-6)+(+2)]-(-5)$

v) $-[+(-7)]-(+3)$ w) $+\{-5-[-3-(-2)]\}$ x) $-(+1)+(-[-4+2])$

3. Calculate the following questions containing multiplication and division

a) $+6\times(-3)$ b) $-7\times(-2)$ c) $+4\times(+5)$

d) $-8\times(+6)$ e) $+9\times(-4)$ f) $+12\div(-3)$

g) $-15\div(+5)$ h) $+18\div(+6)$ i) $-16\div(-4)$

j) $+21\div(-7)$ k) $+2\times(-3)\times(-4)$ l) $-5\times(-2)\div(+1)$

m) $+6\times-1\times2\times(-2)$ n) $-3\times(-3)\times(-2)$ o) $+10\div(-2)\times(-5)$

4. Calculate the following (watch out for BODMAS)

a) $-4+8\times-2$ b) $+6-12\div-3$ c) $-3\times(-4)+7$

d) $-18\div6-(-2)$ e) $+9-5\times-2$ f) $-7+10\div-5$

g) $-12\div-3+8$ h) $+3\times-2-5$ i) $-6+-4\times2$

j) $+6-[(-2)+(-3\times-1)]$ k) $-\{[-3+7]\times(-2)\}$

EXPANDING AND COLLECTING

To expand in algebra means to multiply the term outside the brackets by each term inside the brackets. This process can also be called distribution, as you are distributing the outside term to every term inside.

Worked examples.

a) $2(r+5)$
$= 2 \times r + 2 \times 5$
$= 2r + 10$

b) $3(f-4)$
$= 3 \times f + 3 \times -4$
$= 3f - 12$

c) $5(2+r)$
$= 5 \times 2 + 5 \times r$
$= 10 + 5r$

With your teacher.

a) $4(x-2)$

b) $3(a+1)$

c) $-2(5-y)$

Only 'like terms' can be added or subtracted. 'Like terms' have the same combination of letters, to the same power. The order of letters is not important.

These are **like terms**	These are **not like terms**
$2a + 4a$	$3x + 3x^2$
$3zc + 8zc$	$5a + 5b$
$5a^2 + -2a^2$	$x^{2y} + xy^2$
$x^2y + 4x^2y$	$3mn + 3m^2n$
$-3xy^2 + 8xy^2$	$7 + X$
$6 + -2$	$m^3n + mn^3$
$2m^3n + 9nm^3$	$9xy + 9xyz$
$a^2b + -ba^2$	$x + 1$

Steps to adding or subtracting algebraic terms:

1. Identify like terms (same combination of letters, to the same powers)
2. Rewrite expression so the like terms are grouped together.
3. Add or subtract the coefficients (keep the variables the same)
4. Write the simplified result.
5. Repeat if there are more like terms. Keep grouping and simplifying until there are no more like terms to combine.

Worked examples.

$7p + 4y - 3 - 2p + 6$

$= 7p - 2p + 4y - 3 + 6$

$= 5p + 4y + 3$

$3(2a + 5) + 2(a - 4)$

$= 3 \times 2a + 3 \times 5a + 2 \times a + 2 \times -4$

$= 6a + 15 + 2a - 8$

$= 6a + 2a + 15 - 8$

$= 8a + 7$

With your teacher.

a) $6t + 2 - 2t - 5$

b) $5a - 3a + 4$

c) $4k + 7y - 1 - 6k + 2$

d) $2(p + 4) - 3$

e) $4(a - 1) + 2(3a + 2)$

f) $3(2s + 5) - 2(-s + 3)$

g) $4(2d^2 - 3d + 1) - 3(d^2 - 4d - 2)$

h) $\frac{1}{2}a + 4\left(\frac{3}{4}a - \frac{1}{4}a\right) - a$

Try it yourself (EXPANDING AND COLLECTING).

1. Expand each of the following.

a) $3(a+5)$ b) $2(a+3)$ c) $8(q-3)$

d) $5(b-4)$ e) $4(d-1)$ f) $-2(e+5)$

g) $-3(c+2)$ h) $-2(k-4)$ i) $-6(h-2)$

j) $-2(d-5)$ k) $8(4c+3)$ l) $7(2z-5)$

m) $-7(3v+1)$ n) $3(-u-2)$ o) $3(-u-2)$

2. Simplify the following by collecting like terms

a) $4a+7a$ b) $9b-3b$ c) $5c+2c-c$

d) $-3d+6d$ e) $8e-2e+5e$ f) $6g+(-3g)+g$

g) $1.5h+0.5h$ h) $-2j^2-3j^2+j^2$ i) $2ph^2+4ph^2$

j) $7kmn+2nkm-3nmk$ k) $3a^2+4a^3-2a^2+5a^3$

l) $0.5p^4+1.5p^4-0.25p^4$ m) $-r^3-r+3r^3-2r$

n) $2rp^4p^2+p^2p^4r$ o) $-0prt+0prt+1rpt$

3. Expand and simplify the following:

a) $2(a+3)+4a$ b) $3(b-2)-b$ c) $3(f+2)-5f$

d) $-3(h-4)+h$ e) $-4(c+1)+2c$ f) $5(d-6)+d$

g) $-2(e-3)-4e$ h) $4(j^4+1.5)-2j^4$ i) $6g-2(g-1)$

j) $3t^4+2(-t^4+4)$ k) $5n^3-3(n^3-6)$ l) $2(y^5-2)+3(y^5+1)$

m) $-2(mr^5+5)-(mr^5-2)$ n) $-3(w^5-2)-2(w^5+1)$

4. Liam is buying school supplies. He buys 3 packs, and each pack contains 2 pens and 1 notebook. He also buys 2 extra pens. The total cost can be represented by the expression **3(2p + n) + 2p**. Expand and simplify the expression.

5. Emma is organising gift bags for a fundraiser. Each bag contains 3 stickers, 2 keychains, and 1 badge. She prepares 4 gift bags and then adds 3 more keychains and 2 more badges separately. The total cost could be represented by the expression: **4(3s + 2k + b) + 3k + 2b**. Expand and simplify the expression.

SOLVING LINEAR EQUATIONS

An equation shows that two expressions are equal, just like the two sides of a balanced scale. When the left side has the same value as the right side, the equation is balanced.

When **solving an equation**, the goal is to isolate the variable (often x, but any letter or symbol can be used) by performing the same operation on both sides.

Worked example. Imagine you have two small buckets hanging from a scale. One bucket has x coins plus 3 more (x + 3), and the other has exactly 7 coins (7). If the scale is balanced, then both sides weigh the same: $x + 3 = 7$

To find out how many coins are in the first bucket (x), remove 3 from both sides:

$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 \qquad \leftarrow \text{Remove 3 from both sides} \\ x &= 4 \end{aligned}$$

Therefore, the letter x represents 4 coins.

To isolate the variable, you need to remove everything on that side that isn't the variable you are trying to isolate. We remove terms by doing the opposite operation.

Opposite operations

Addition (+) and subtraction (–) are opposites

Multiplication (×) and division (÷) are opposites

Worked examples.

a)
$$\begin{aligned} h + 2 &= 5 \\ h + 2 - 2 &= 5 - 2 \\ h &= 3 \end{aligned}$$

b)
$$\begin{aligned} j - 5 &= 12 \\ j - 5 + 5 &= 12 + 5 \\ j &= 17 \end{aligned}$$

c)
$$\begin{aligned} 3t &= 12 \\ 3t \div 3 &= 12 \div 3 \\ t &= 4 \end{aligned}$$

d)
$$\begin{aligned} {}^{r}/_{10} &= 2 \\ {}^{r}/_{10} \times 10 &= 2 \times 10 \\ r &= 20 \end{aligned}$$

With your teacher.

a) $x + 4 = 9$ b) $x + 5 = 12$ c) $x - 2 = 7$

d) $x - 9 = -1$ e) $3x = 21$ f) $-2x = -10$

g) $-x = 6$ h) $x \div 5 = 6$ i) $\frac{J}{3} = 10$

Solving equations often require more than one step. In this situation, you need to do the **reverse of BODMAS**. Sometimes it helps to write down what order you would normally complete the steps, and then do them backwards.

Worked examples.

a)

$$\begin{aligned} 2r - 10 &= 14 \\ 2r - 10 + 10 &= 14 + 10 \\ 2r &= 24 \\ 2r \div 2 &= 24 \div 2 \\ r &= 12 \end{aligned}$$

b)

$$\begin{aligned} 3a + 5 &= 2a + 12 \\ 3a + 5 - 5 &= 2a + 12 - 5 \\ 3a &= 2a + 7 \\ 3a - 2a &= 2a - 2a + 7 \\ a &= 7 \end{aligned}$$

With your teacher.

a) $2y + 3 = 11$ b) $5e - 4 = 16$

c) ${}^{w}/_{3} - 2 = 4$ d) ${}^{1}/_{4}\, q + 3 = 6$

With your teacher (continued).

e) $2(w - 4) = w + 3$

f) $6n - (2n + 1) = 3n + 7$

g) $^1/_2\,(j + 4) = 3j - 2$

h) $5u - 2 = 3(u + 1)$

Try it yourself (SOLVING LINEAR EQUATIONS).

1. Solve each equation for y.

a) $y + 7 = 12$ b) $y - 4 = 9$ c) $y + 6 = -2$

d) $y - 3 = -7$ e) $5y = 35$ f) $-3y = 12$

g) $-5 = y - 3$ h) $y \div 4 = 6$ i) $y \div -5 = -2$

j) $y - {}^{3}/_{4} = {}^{1}/_{4}$ k) $y \div {}^{1}/_{2} = 10$

2. Solve each equation below for y.

a) $2y + 3 = 9$ b) $-4y + 5 = -7$ c) $-3y - 8 = 10$

d) $-5 + {}^{y}/_{2} = 1$ e) ${}^{y}/_{3} - 2 = 6$ f) $2(y + 3) = 14$

g) $4(y - 1) + 2 = 10$ h) $-3(y - 2) + 1 = -5$ i) $-4 + 2(y + 3) = 10$

j) $3 + 2y + (3y - 4) = 9$ k) $2y + 3 = y + 9$ l) $4y - 5 = 2y + 3$

m) $6y + 2 = 3y + 8$ n) $12y - 3 = 3y + 2y + 4$

3. Rearrange each equation so **y** becomes the subject:

a) $y + 6 = x$ b) $4y = a$ c) $y - 3 = m$

d) $x = y - 8$ e) $z = 7y$ f) $3y + 5 = x$

g) $a = 6y - 4$ h) $t = 2y + 9$ i) $x = 5 - 4y$

j) $z = 4(y + 2) - 3$ k) $x = 3(y - 1) + 2(y + 2)$

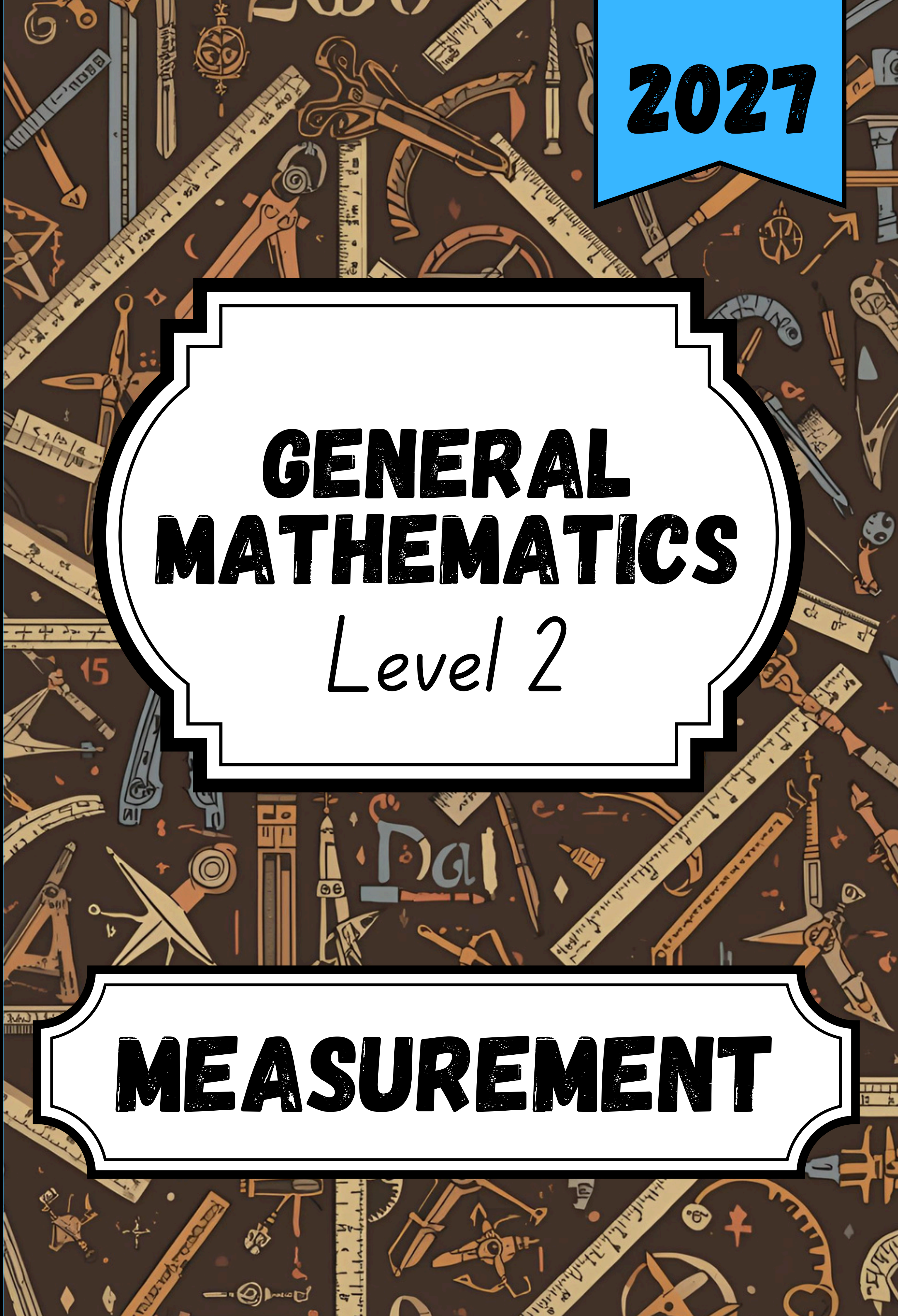
l) $x = 2(y + 4)$ m) $a = 3(y - 5) + 6$

4. Harper buys one notebook and three identical pens. The notebook cost \$3 and she spends \$12 in total. Write and solve an equation to find the cost of each pen.

5. A phone plan has a fixed monthly fee plus charges \$5 for every gigabyte of data. If a \$45 bill included 6GB of data, what was the fixed monthly fee?

6. A plumber charges \$80 for a service call plus \$65 per hour. If the total bill was \$275, how many hours did the plumber work?

7. An Uber trip costs \$4.50 plus \$1.80 per kilometre. If you paid \$21.30, how many kilometres did you travel?



2027

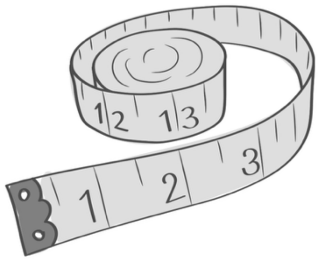
GENERAL MATHEMATICS

Level 2

MEASUREMENT

Part 1

MEASUREMENT



INTRODUCTION TO MEASUREMENT

Measurement is how we describe and compare the size of things in the real world. We use measurement every day - from checking our height, to filling up a petrol tank, to building houses and designing controversial AFL stadiums.

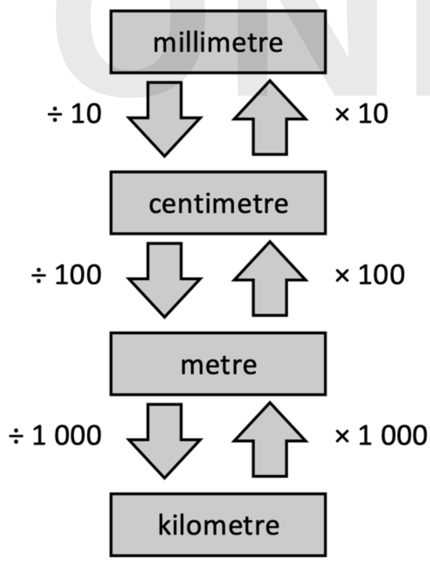
In mathematics, measurement allows us to connect numbers to real objects by using units such as centimetres, metres, and litres.

When solving problems, it's important to **use the same units** for all parts of the question. If measurements are given in different units, convert them first.

Complete with your teacher.

1. Convert the following measurements.

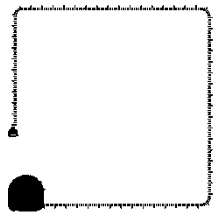
Original	Convert to
100mm	cm
3,506mm	cm
200cm	m
1,346cm	m
2,000m	km
3,506mm	m
210km	m
35m	cm
22.5cm	mm
2.5km	mm



2. Add together the following: 125mm + 35.6cm +1050mm + 1.5m + 27.4cm

PERIMETER

Perimeter is the distance around the outside of a two-dimensional shape. It is a linear measurement, usually measured in millimetres (mm), centimetres (cm), metres (m) or kilometres (km).



To calculate perimeter, we add together the lengths of all the sides of the shape.

In this section we will cover how to find the perimeter of the following shapes:

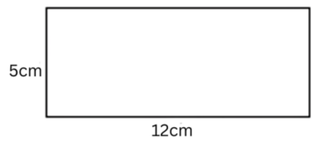
- Rectangles
- Triangles
- Parallelograms
- Circles (including sectors)

Worked Examples. *(rectangles, triangles & parallelogram)*

1. A rectangle has a length of 12 cm and a width of 5 cm.

   $Perimeter = 5 + 5 + 12 + 12$

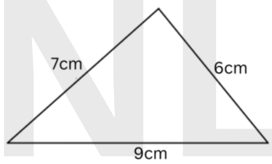
   $P = 34\ cm$



2. A triangle has sides of 6 cm, 7 cm, and 9 cm.

   $Perimeter = 6 + 7 + 9$

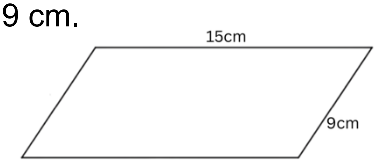
   $P = 22\ cm$



3. A parallelogram has a base of 15 cm and a side length of 9 cm.

   $Perimeter = 15 + 9 + 15 + 9$

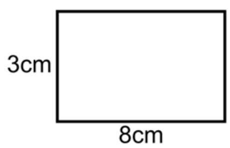
   $P = 48\ cm$



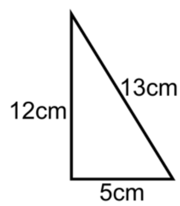
With your teacher.

1. Find the perimeter of the following shapes.

   a) Rectangle with length 8 cm and width 3 cm.

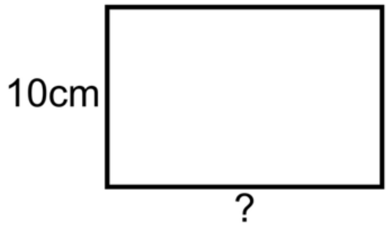


   b) Triangle with side lengths 5 cm, 12 cm, and 13 cm.

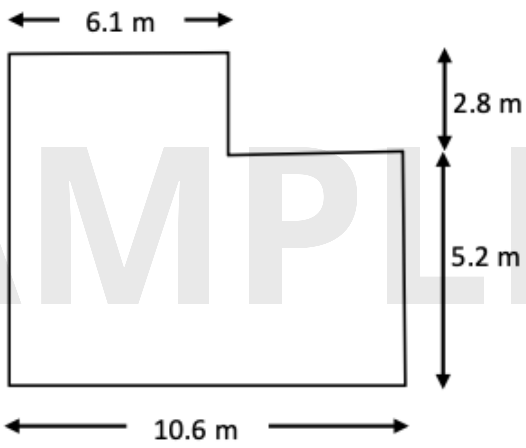


With your teacher.

2. A rectangle has a perimeter of 50 cm and a width of 10 cm. What is its length?



3. Find the perimeter of the following shape:



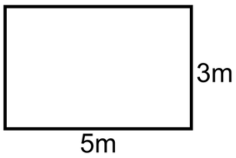
Try it yourself (PERIMETER).

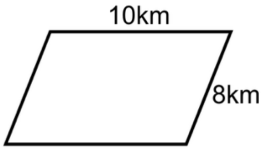
1. Change the followings units. Round to 2 decimal places (2dp) if needed.

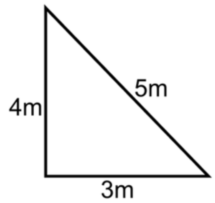
a) 5m in cm	b) 4.63m in cm	c) 5.4m in mm	d) 5.28m in mm
e) 5276m in km	f) 756m in km	g) 1235mm in cm	h) 135mm in cm
i) 5682mm in m	j) 3682mm in m	k) 39cm in mm	l) 820cm in mm
m) 350cm in m	n) 1274cm in m	o) 3.5km in m	p) 0.85km in m

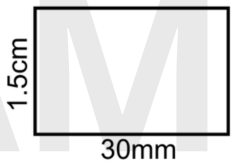
2. Arrange each of the following in ascending order (smallest to biggest):
   1m, 3.5cm, 1cm, 450mm, 25mm, 2370mm, 320cm, 2.2m, 5mm

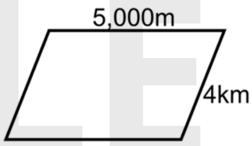
3. Find perimeter of the following shapes.

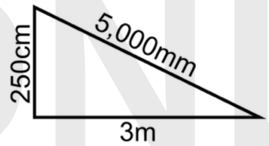
a)


b)


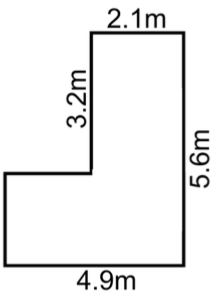
c)


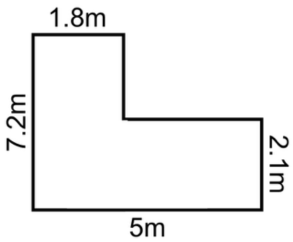
d)


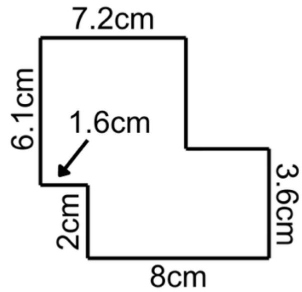
e)


f)


4. Find the perimeter of the following shapes.

a)


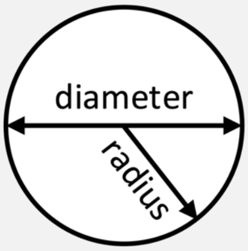
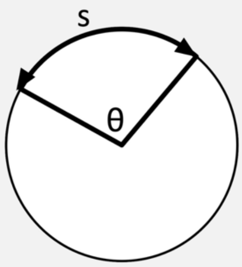
b)


c)


5. A triangle has two equal sides of 8cm and a base of 60mm. Find its perimeter.

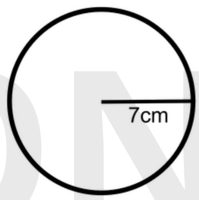
6. A builder is constructing a deck that is 3.2m wide. How many decking boards will be used if each decking board is 70mm wide and a 6mm gap is to be left between each board?

7. A carpet costs $140 per metre length and is cut from a roll that is 4m wide. Find the lowest cost of carpeting a hall which measures 11500mm x 8400mm.

<table>
<tr><th colspan="3">Perimeter with circles (circumference)</th></tr>
<tr><td>
</td><td>Full circumference
$C = \pi d$
or
$C = 2\pi r$</td><td>Where:
d = diameter
r = radius</td></tr>
<tr><td>
</td><td>Arc length
$S = \pi d \times \frac{\theta}{360}$
or
$S = 2\pi r \times \frac{\theta}{360}$</td><td>Perimeter of a sector
$P_s = S + 2r$</td></tr>
</table>

Worked examples.

1. Find the circumference of a circle with a radius of 7cm.

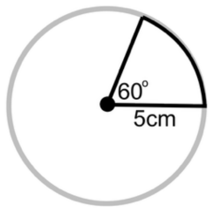


$$C = 2\pi r$$

$$C = 2 \times \pi \times 7$$

$$C = 43.98\text{cm}$$

2. Find the perimeter of a sector that has radius of 5cm and an included angle of 60°.



$$P_s = \left[2\pi r \times \frac{\theta}{360}\right] + 2r$$

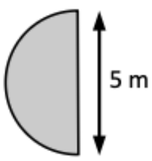
$$P_s = \left[2 \times \pi \times 5 \times \frac{60}{360}\right] + 2 \times 5$$

$$P_s = 15.24\text{cm}$$

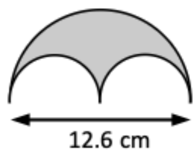
With your teacher.

Find the perimeter of the following shapes.

a)

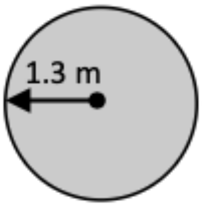


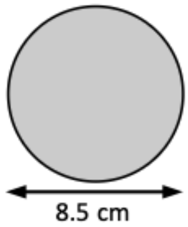
b)

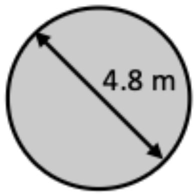


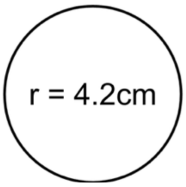
Try it yourself (CIRCUMFERENCE).

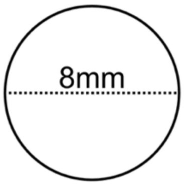
1. Find the circumference of the following circles.

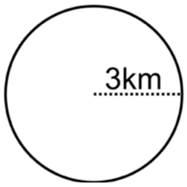
a) 

b) 

c) 

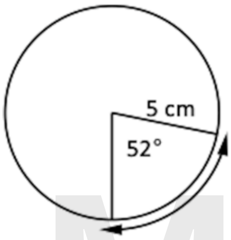
d) 

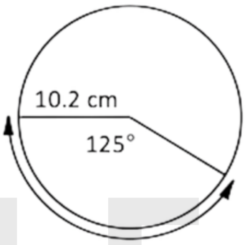
e) 

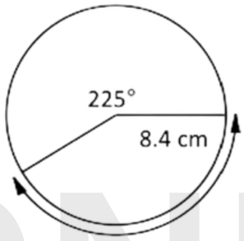
f) 

2. Find the circumference of a circle with diameter 12 cm.

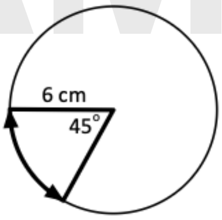
3. Find the arc length of the following:

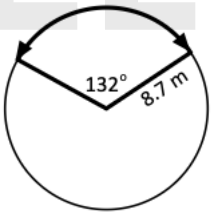
a) 

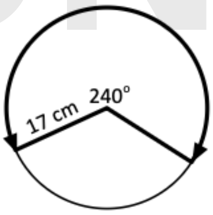
b) 

c) 

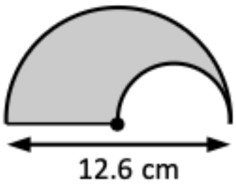
4. Find the perimeter of the following sectors.

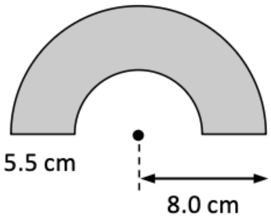
a) 

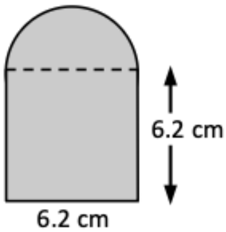
b) 

c) 

5. Find the perimeter of the following shapes.

a) 

b) 

c) 

6. A windscreen wiper has a length of 42cm and swings through an arc of 125°. Find the distance moved by the tip of the blade with each swing. (arc length)

7. A wrist-watch has a diameter of 3cm. The second hand has a length of 1.4cm. How far does the tip of the second hand move in 5 seconds?

8. A child is on a swing which has ropes of length 3m, moving through an angle of 35°. How far has the child travelled after 1 minute if a complete swing (back and forwards) is made every 4 seconds?

2027

GENERAL MATHEMATICS

Level 2

DATA

STANDARD DEVIATION (SD)

Standard deviation is a measure of spread. It tells us, on average, roughly how far the data values are from the mean.

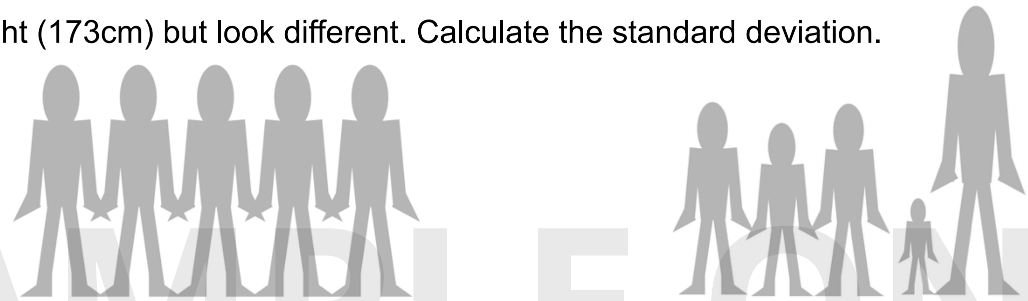
- A small SD means the data is tightly clustered around the mean (consistent).
- A large SD means the data is more spread out (less consistent).

Standard deviation formula (from a table)

$$SD = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

Worked example.

Two basketball teams consist of 5 people each. Both teams have the same average height (173cm) but look different. Calculate the standard deviation.



Team A
Smaller standard deviation.
All values (heights) are closer to the average value (height)

H = 172, 175, 174, 172, 172 (cm)

Team B
Larger standard deviation.
Individual values (heights) are further away from the average height.

H = 170, 165, 170, 140, 220 (cm)

Calculating standard deviation for team A.

Height (x)	Frequency (f)	x^2	fx	fx^2
172	1	29,584	172	29,584
175	1	30,625	175	30,625
174	1	30,276	174	30,276
172	1	29,584	172	29,584
172	1	29,584	172	29,584
Σ	**5**	**149,653**	**865**	**149,653**

$$SD = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

$$SD = \sqrt{\frac{149{,}653}{5} - \left(\frac{865}{5}\right)^2}$$

$$\boldsymbol{SD = 1.26}$$

<u>Interpretation in context:</u> The standard deviation of 1.26 means that, on average, most player's heights are 1.26cm away from the team's average height (173cm).

With your teacher.

Calculate the standard deviation of team B.

Height (x)	Frequency (f)	x^2	fx	fx^2
170	2			
165	1			
140	1			
220	1			
Σ	**5**			

$$SD = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

In practice, standard deviation is usually calculated with technology.

You can use a CAS calculator, or an online calculator like Desmos or GeoGebra.

Free online calculators for standard deviation
Desmos scientific https://www.desmos.com/scientific → Function → stdevp → stdevp(172,175,174,172,172) = 1.26
GeoGebra Scientific calculator https://www.geogebra.org/scientific → Keyboard (bottom left) → f(x) → stdevp(172,175,174,172,172) = 1.26

Limitations of standard deviation

Standard deviation is a useful measure of spread, but only as part of a bigger picture.

- SD is heavily influenced by outliers and extreme values
- SD works best with a symmetric distribution; it may be misleading when used on skewed data.
- SD only makes sense when considered alongside the mean, not on its own.

Try it yourself (STANDARD DEVIATION).

1. Calculate the standard deviations for each of the below tables.

a)

x	f	x^2	fx	fx^2
20	5			
25	10			
35	6			
40	7			
Σ				

b)

x	f	x^2	fx	fx^2
4	12			
5	14			
7	6			
8	2			
Σ				

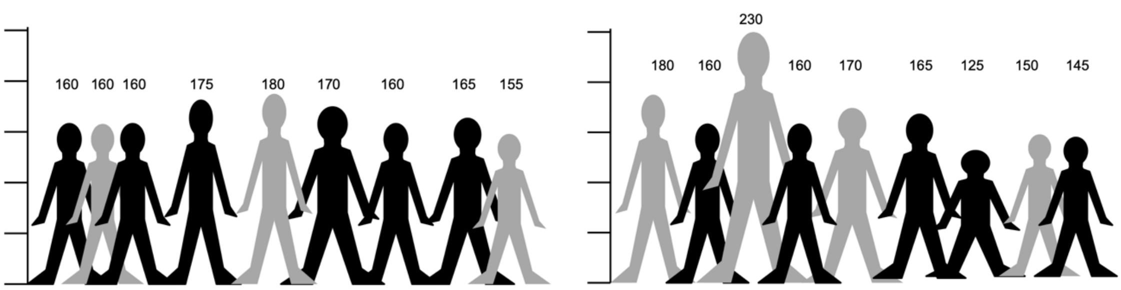
2. Use an online calculator like Desmos or GeoGebra to find the SD of this data:
   a) 12, 15, 14, 12, 11, 16, 17, 13, 18, 15, 15, 16
   b) 42.5, 47, 49.3, 50.55, 48.2, 45.7, 51.6
   c) 100, 112, 110, 101, 105, 113, 110, 110, 110, 42

3. The table below shows the number of jellybeans in randomly chosen packets produced on an automatic packing machine. The company insists the process is consistent, with a standard deviation of less than 1 jellybean per packet.

# per pack (x)	f	x^2	fx	fx^2
20	5			
25	10			
30	4			
35	6			
40	7			
Σ				

   a) Complete the table and calculate the mean and standard deviation.
   b) Is the company correct in saying the standard deviation is less than 1?
   c) Comment on any limitations of using SD alone in this context.

4. For each of group of people pictured below (group A on the left, B on the right):



   a) Find the mean, mode, range and standard deviation for both groups.
   b) If the tall person from group B was to stand in group A, how would the statistics of group A be affected?

INVESTIGATION 2 – FACTORY WAGES

A bitter dispute has broken out at a local factory.

- The CEO is running advertising claiming that employees are generously paid, with an average salary of over $40,000 per year.
- The Union, representing the factory workers, argues that this is dishonest - most workers are poorly paid, and the CEO is using misleading statistics to make the factory look good.

Both sides are determined to prove their case using the same set of salary data. It's your job to analyse the data, and then sharpen your pitchforks - either as the CEO's marketing team or the Union's campaign team.

PART 1 – crunch the numbers *(part 1 can be done individually or in pairs)*

The salaries for all employees are listed below.

Employee	Salary $'000 ($x$)	Frequency (f)	x^2	fx	fx^2
Worker	$15	30			
Supervisor	$18	4			
Manager	$75	6			
CEO	$850	1			
Σ					

Calculate the mean, mode, range, and standard deviation of the salaries.

Mean	SD	Mode
		Range

INVESTIGATION 2 (continued)

Is the CEO lying about the average salary being above $40,000?

Describe the distribution of the data (modality, shape, skew, centre, spread)

If the CEO's salary is taken out, how do the mean, mode, range and SD change?

Employee	Salary $'000 (x)	Frequency (f)	x^2	fx	fx^2
Worker	$15	30			
Supervisor	$18	4			
Manager	$75	6			
Σ					

Mean	SD	Mode
		Range

Why do the two sides reach such different conclusions from exactly the same data?

Why does the CEO's salary have such a big effect on the mean and standard deviation?

INVESTIGATION 2 (continued)

PART 2 – Pick your side *(part 2 can be done in groups, or individually)*

Each student in the class will be assigned to either the CEO's marketing team, or the Union's campaign team. Each group will creating a media campaign using the statistics that best support their assigned side (CEO or Union). You may make:

- A poster, infographic, brochure or slideshow (try *canva.com*)
- A TV, radio, or social media advertisement (Instagram, TikTok, etc)

Brainstorm / sketch / planning / work paper for your media campaign

INVESTIGATION 2 (continued)

PART 3 – Pitch and compare

Each group should present their media campaign to the class. Discuss which student/group was able to use the statistics most convincingly. Who did you believe?

Notes from viewing media campaigns.

INVESTIGATION 2 (continued)

PART 4 – Class reflection

Once everyone has presented, discuss the following prompts as a class.

- Which statistics did you think were most misleading?

- If the CEO hired his best friend as a Co-CEO and paid him the same salary, what impact do you think this would have on the statistics?

- How easy is it to cherry-pick data to make a point?

- Can you think of a real-life situation where someone has selected misleading statistics to further their point? (You can't use Trump, he's too easy)

2027

GENERAL MATHEMATICS

Level 2

LINEAR

LINE OF BEST FIT

When we have a scatterplot showing a relationship between two variables, we can summarise the relationship using a **line of best fit.** This line is not drawn through all the points - it is placed so that it best represents the overall trend of the data.

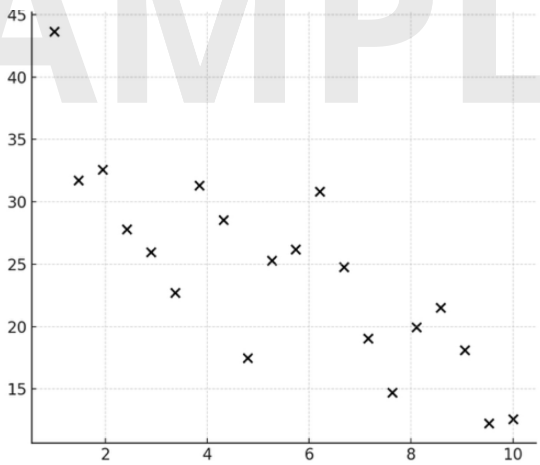
Fitting a line of best fit can be done by hand or using technology.

Line of best fit - by hand

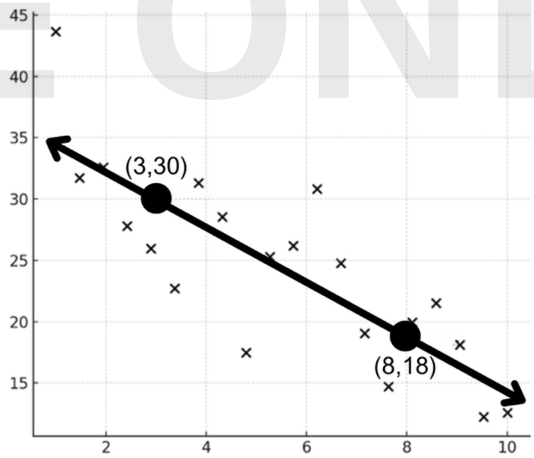
- Plot the scatterplot.
- Use a ruler to draw a straight line that seems to follow the trend of the data.
- Try to “balance” the points so there are roughly as many above the line as below.
- Choose two points **from the line you’ve just drawn** (they don’t have to be any of the original points) and use them to calculate the equation y = mx + c.

Worked example.

Step 1: Plot the scatterplot



Step 2: Fit the line



Step 3: Use two points on the line **(8,18)** and **(3,30)** to determine the equation.

Step 1: Find the gradient $\rightarrow$ $m = \frac{18-30}{8-3} = \frac{-12}{5} = -2.4$

Step 2: Sub points into formula $\rightarrow$ $y - y_1 = m(x - x_1)$

$$y - 18 = -2.4(x - 8)$$

$$y - 18 = -2.4x + 19.2$$

$$y = -2.4x + 37.2$$

Once we have a line of best fit – we can use it to make predictions.

Making predictions using line of best fit (Interpolation vs Extrapolation)

We can use a line of best fit to make predictions. However, how reliable those predictions are depends on where we are making the predictions.

Interpolation	Extrapolation
Making predictions **within** the range of data we have. Because the line of best fit was created from data in this range, the prediction is usually **reliable**. **Example:** If data was collected for students aged **10 to 18**, predicting the height of a **15-year-old** using the line of best fit is **interpolation**. It is within the known data range.	Making predictions **outside** the range of available data. Since we don't know if the same pattern continues outside the data range, extrapolation is **less reliable**. **Example:** Using the same height data to predict how tall someone will be at **age 25** is **extrapolation** — the trend may not continue once people stop growing.

Worked example.

The hours of study and average exam score for a group of students is recorded.

Study time (hours)	1	2	3	4	5	6
Exam score (%)	45	55	60	68	73	82

The line of best fit is y = 7x + 38 (x = hours studied, y = average exam score %)

a) Predict the score for a student who studied for 4.5 hours.

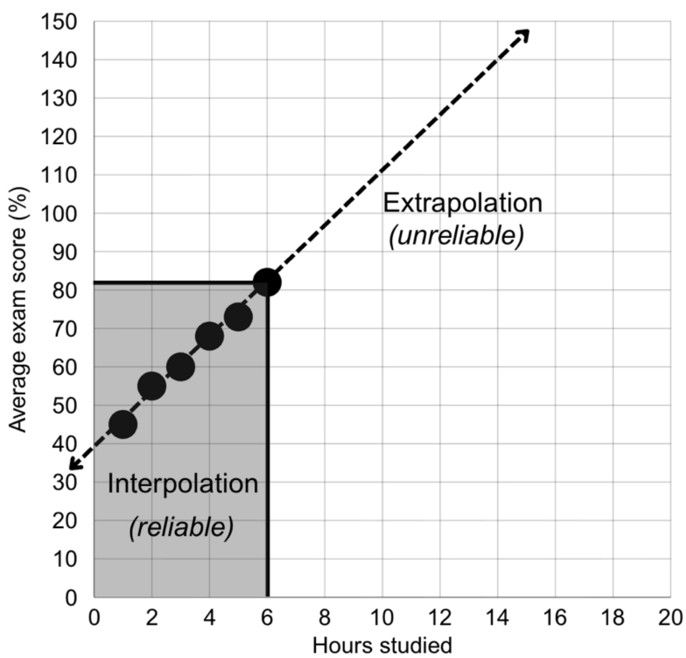
$y = 7(4.5) + 38 = 69.5\%$

This is **interpolation.** It is within the given data (1–6 hours).

b) Predict the score for a student who studied for **10 hours.**

$y = 7(10) + 38 = 108\%$

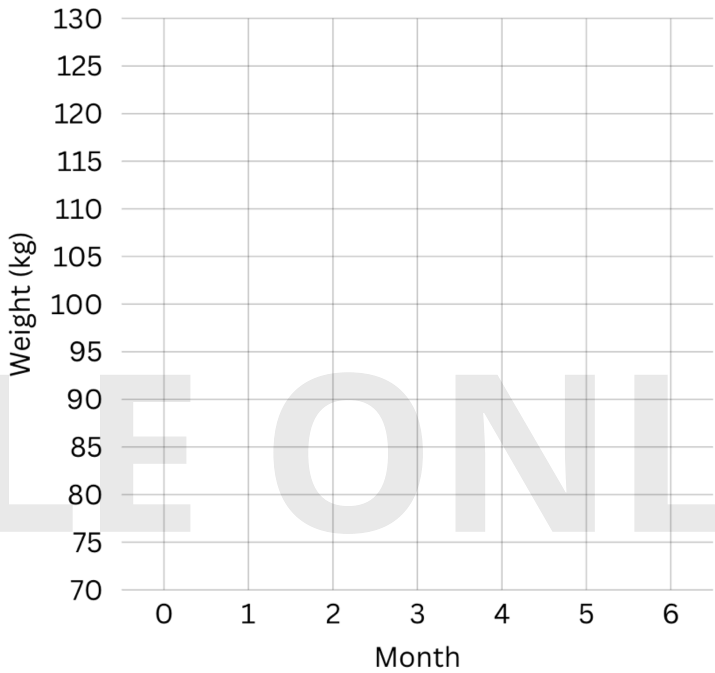
This is **extrapolation** and the prediction doesn't make sense (over 100%!).



With your teacher.

1. The director of a weight loss clinic asks clients to record their weight each month. The below table details the data for a client.
   a) Plot a scatter plot of the data.
   b) Fit the trend line ‘by eye’ and use the two-point method to find its equation.
   c) The client has a big event in 10 weeks (2.5 months) and wants to know what he will weigh. Use the graph to predict his weight at this time, and then predict the weight again using your equation. Are the predictions different? Why?
   d) Predict the client’s weight after 2 years. What do you notice?

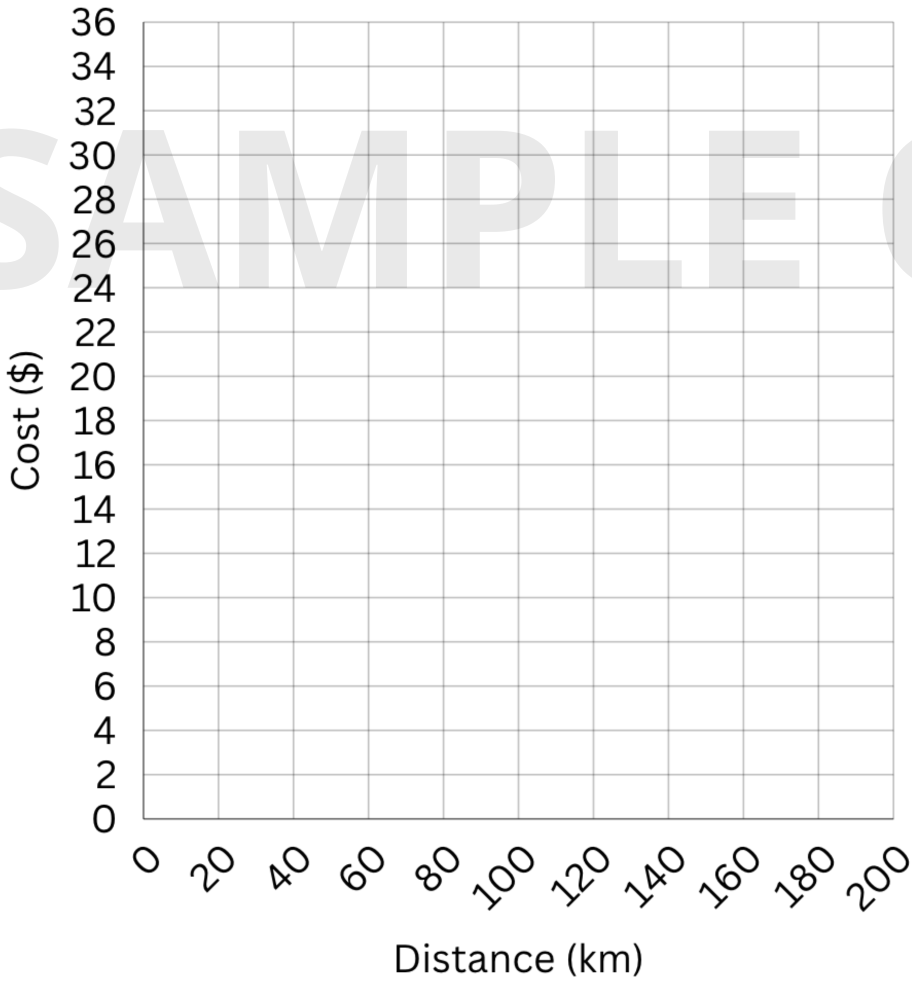
Month	Weight (kg)
0	128
1	115
2	118
3	102
4	99
5	87
6	82



With your teacher.

2. The adult fares for bus travel from Hobart are given by the table shown. Plot a scatterplot of the data then use the two-point method to find the equation of the trend line. Express the relationship in terms of distance 'd' and fare 'f'. Give your answer in exact form, using fractions rather than decimals where possible.

Trip	Distance (km)	Cost ($)
Hobart - Triabunna	72	20.00
Hobart - Bicheno	178	36.00
Hobart - New Norfolk	26	8.00
Hobart - Geeveston	60	13.00
Hobart - Port Arthur	95	23.00
Hobart - Launceston	198	36.00



Line of best fit – using technology (regression)

Finding a line of best fit using technology is known as

"regression" or "linear regression"

It is far more accurate than fitting a line by hand.

There are many ways to use technology to calculate regression.

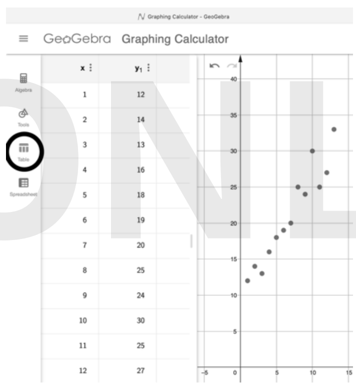
In this workbook we will use two free websites; GeoGebra and Desmos.

Worked example using GeoGebra → https://www.geogebra.org/graphing

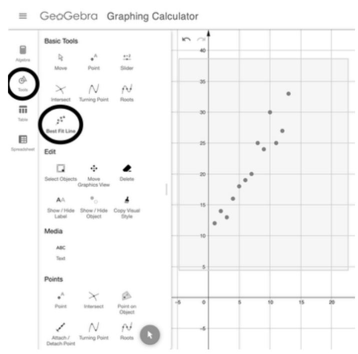
Eric recorded the temperature in Melbourne over 13 days. He wants to predict the temperature for a possible beach day in 30 days (from day 1). The temperatures are recorded below, find the line of best fit and predict the temperature on day 30.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
°C	12	14	13	16	18	19	20	25	24	30	25	27	33

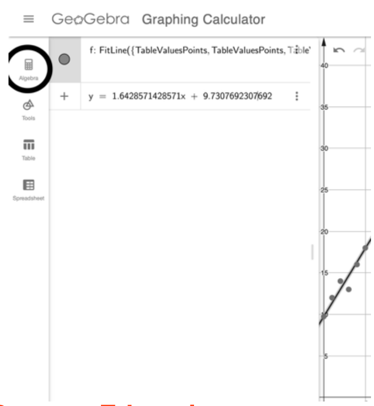
Step 1: Enter the data into a **table** in GeoGebra's graphing calculator



Step 2: Click tools → best fit line → highlight all your points. GeoGebra will draw a line of best fit through your points.



Step 3: To view the equation of the line of best fit → click algebra → $y = 1.6428x + 9.7308$



Interpreting in context. This equation shows that the temperature starts at nearly 10°C and increases by around 1.6°C each day. In 30 days the temperature should be nearly 59°C!

A line of best fit allows us to make predictions. But there are limitations, and not all predictions are reliable.

Interpolation = predictions made within the data range (eg 10 days). Considered more reliable.

Extrapolation = predictions outside the data range, (eg 30 days). Considered much less reliable.

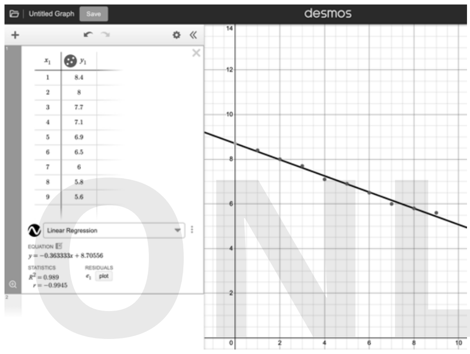
Worked example using Desmos → www.desmos.com/calculator

Elly often feels more tired after days when she uses screens all day. She decides to investigate if screen time during the day impacts her sleep of a night.

Over 9 days, Elly experimented with her screen time, limiting her time to 1 hour on the first day and increasing by an hour each day until she was on a screen for 9 hours per day. She used her phone to monitor the number of hours sleep she got that night and recorded the results in the table below.

Screen time (hours)	1	2	3	4	5	6	7	8	9
Sleep time (hours)	8.4	8.0	7.7	7.1	6.9	6.5	6.0	5.8	5.6

Step 1: In Desmos' graphing calculator, click the **+** in the top left-hand corner and add a table. Enter the data above into the table using screen time as (x) and sleep time as (y).



Step 2: Click the 'add regression' button near the top left of your table. (it looks like this →)

Step 3: Copy the equation that appears.

In this example it is $y = -0.3633x + 8.7056$

With your teacher.

Interpret in context. *Interpret the equation in relation to this example. What do the gradient and y intercept mean? Is there a relationship between screens and sleep?*

Make predictions.

How much sleep would Elly get with 5.5 hours of screen time?

What about 25 hours of screen time?

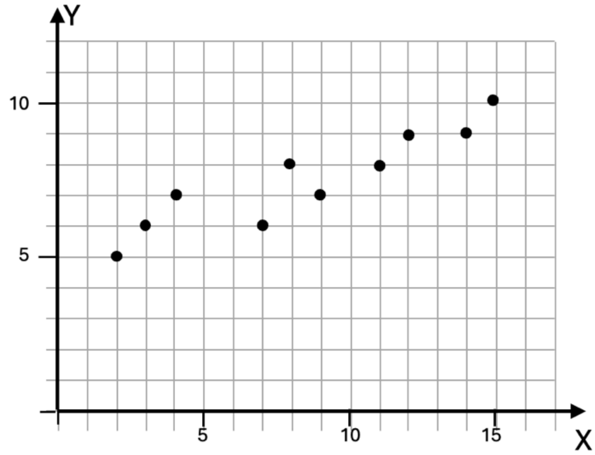
If Elly wants exactly 7 hours sleep what is her screen limit?

At what point in the example does extrapolation become unreliable?

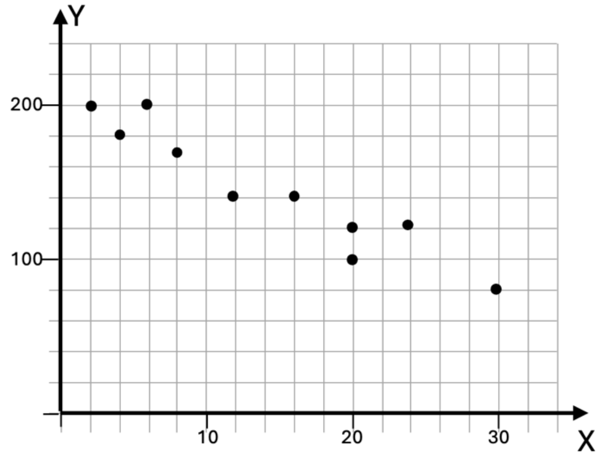
Try it yourself (LINE OF BEST FIT).

1. For each of the following scatter graphs mark in the line of best fit by eye then find the equation of the line of best fit.

a)



b)



2. Calculate the line of best fit for the following data by hand. (graph paper below).

a)

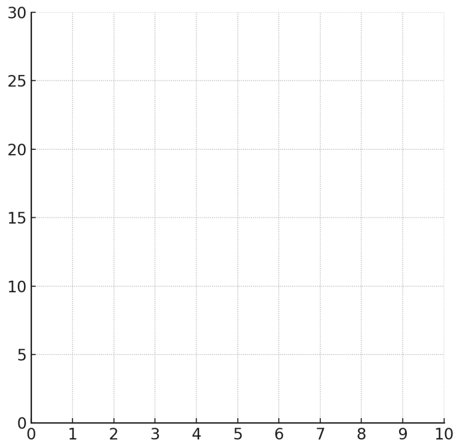
X	1	2	3	4	5	6	7	8	9
Y	8	11	14	16	19	22	24	27	29

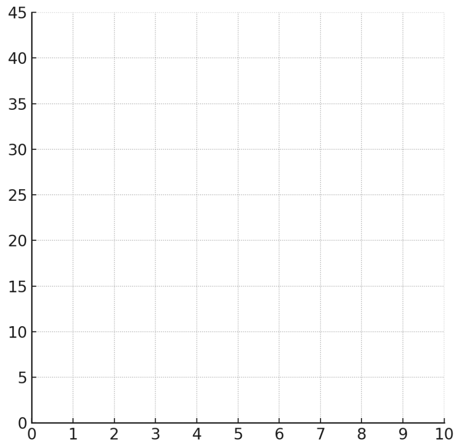
b)

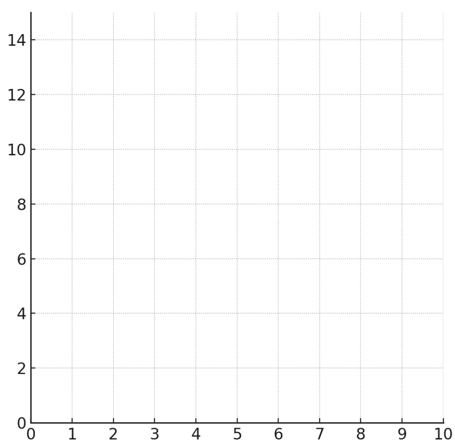
X	1	2	3	4	5	6	7	8	9
Y	44	42	40	38	35	33	30	28	26

c)

X	1	2	3	4	5	6	7	8	9
Y	5	9	7	11	10	13	12	14	15







3. Calculate the linear regression for the following data sets using technology

a)

X	1	2	3	4	5	6	7	8	9
Y	6	8	10	11	14	15	18	19	21

Try it yourself (LINE OF BEST FIT cont).

Question 3 continued.

b)

X	1	2	3	4	5	6	7	8	9
Y	28	26	25	22	20	18	16	14	12

c)

X	1	2	3	4	5	6	7	8	9
Y	19	17	16	18	15	13	14	12	11

4. A student measures the length of a spring as weights are added to it. She produces the results shown.
   a) Determine the relationship between the length of the spring 'L' and the weight attached 'w'.
   b) Use interpolation to predict the length of the string when 300g of weight is added.
   c) Use extrapolation to predict the weight on the spring if the length of the string is 55cm.

Weight (g)	Length (cm)
0	25
200	28
400	30.5
600	34
800	37.5
1000	40
1200	43
1400	46

5. Sometimes, a relationship can appear *strongly correlated* even when they have no relationship at all. Consider this example of pirate population over time.

Women in Leadership ('000s)	Number of Pirates
50	35,000
100	30,000
250	25,000
400	20,000
500	15,000
700	10,000
800	5,000
1,000	3,000

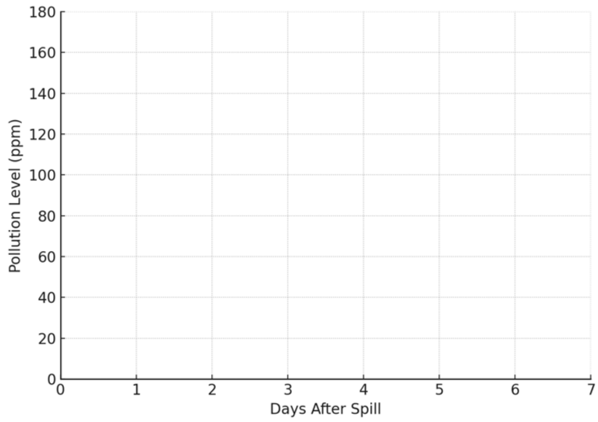
   a) Find the linear regression of the above data. Use the number of women in leadership roles as (X) and the number of pirates as (Y).
   b) Interpret the equation in context of this example.
   c) Predict the number of women in leadership if pirates went fully extinct.
   d) According to your model, how many pirates would have been in the world when no women were allowed to be in leadership?
   e) Have women in leadership caused the decline in pirates? Why or why not?
   f) Can you think of another example where there is a strong correlation between two different things, but they have nothing to do with each other?

Try it yourself (LINE OF BEST FIT cont).

6. Some pollution has been spilled into a river. The table below shows how the pollution level changes over time.

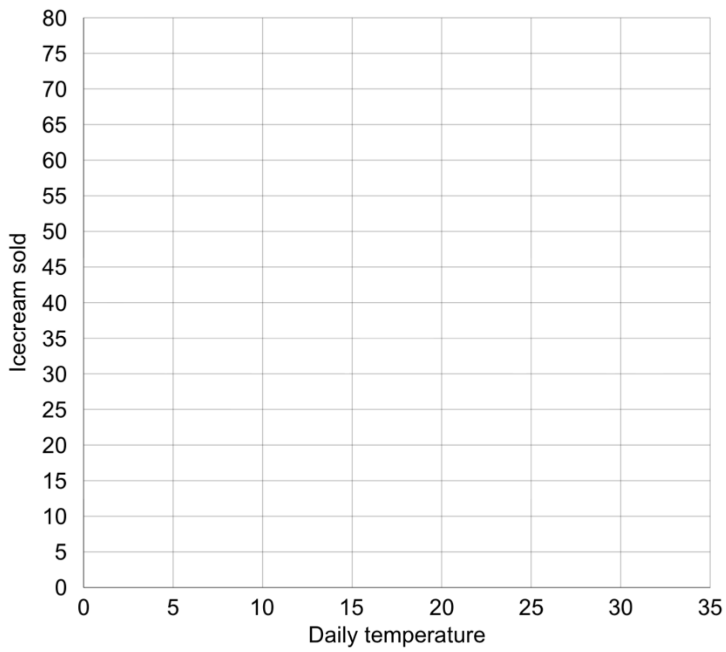
Number of days after spill	0	1	2	3	4	5	6	7
Pollution level (ppm)	160	140	122	99	82	60	45	23

a) Plot the data on a scatter graph.
b) State in words what the graph shows.
c) Mark in the straight line of best fit by eye, then determine the relationship between the level of pollution 'P', and the number of days after the spill 'n'.



7. The operator of an ice-cream truck is interested in finding out how the number of ice-creams that he sells depends upon the temperature of the day. He collects the data and displays it in a table.

a) Graph the data (by hand or using technology)
b) State in words what the graph shows.
c) Find the relationship between the number of ice-creams sold 'N' and the daily temperature 'T'
d) Interpret the y intercept and gradient in context.
e) Predict the number of ice-creams sold when the temperature is 45°C. Is this a reliable prediction? Is the answer realistic? Why/not?

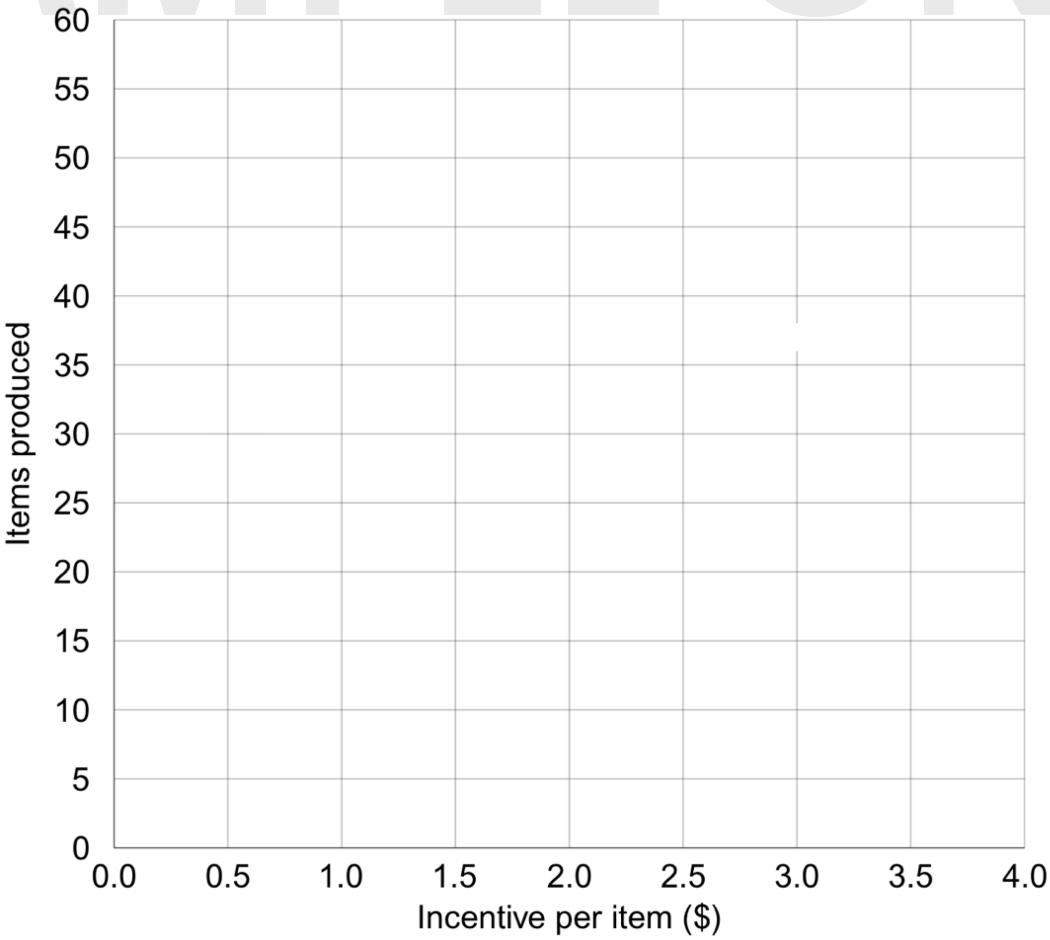


Daily temperature °C	Number of icecreams sold
15	40
18	48
20	56
21	52
24	59
24	63
29	72
34	78

Try it yourself (LINE OF BEST FIT cont).

8. A manufacturer is keen to maximise the output of her workers. The table details the results of a trial in which she offered different monetary incentive to 8 production workers as reward for improved output.
   a) Plot the data on a scatter graph
   b) State in words what the graph shows.
   c) Determine the linear equation of the data.
   d) Interpret the y intercept and gradient in context.
   e) When do you think extrapolation would become unreliable?

Incentive per item	Items produced daily
\$ 0	40
\$ 1.00	43
\$ 1.50	48
\$ 2.00	45
\$ 2.50	49
\$ 3.00	53
\$ 3.50	52
\$ 4.00	56



2027

GENERAL MATHEMATICS

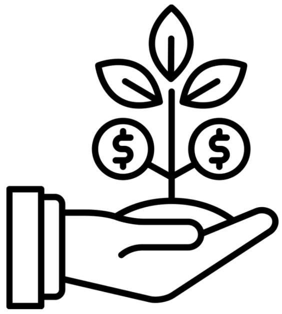
Level 2

MONEY

INVESTING

Investing is a way of earning money by putting your existing money to work.

People invest to build wealth, save for the future, or reach financial goals - like buying a house, retiring comfortably, or simply having more freedom with money.



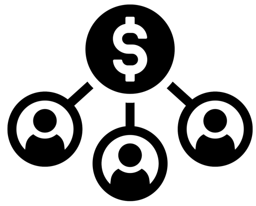
Things to Keep in Mind:

- All investments come with some risk - values can go up or down
- The higher the potential return, the higher the risk tends to be.

Investing is not just for the wealthy - even small amounts invested early can grow significantly over time.

Common types of investments
Real Estate Buying property (like a house or unit) can be an investment. You can: • Earn rental income from tenants • Make a capital gain (the property increases in value) **Shares (the Stock Market)** When you buy shares, you own a small piece of a company. You can: • Earn money when the company pays out dividends (profits to shareholders) • Make a capital gain if the value of the share increases **Interest-Earning Accounts** Placing your money in a savings account, term deposit, or bond means the bank pays you interest - a small amount of money for letting them use your funds. **Collectables** Items like art, vintage toys, sneakers, coins, or trading cards can rise in value over time, especially if they become rare or highly sought after. These are higher risk and less predictable than other types of investments. **Managed Funds and ETFs** These pool your money with other investors to buy a mix of shares, property, or other assets. They're often chosen for diversification (spreading risk).

SHARES AND THE STOCK MARKET



When you buy shares you are buying a small piece of a company.

This makes you a shareholder, meaning you own part of the business - even if it's just a tiny slice.

Shares are bought and sold on the share market (also called the stock market), where prices go up and down depending on how companies are performing and what people think they're worth.

Investing in shares can be a good way to grow your money over time, but it also comes with risk. Share prices can go up or down, so it's important to understand what you're investing in.

🛒 How Do You Buy and Sell Shares?

You can buy and sell shares through an online trading platform.

Many banks in Australia (like CommBank, NAB, Westpac or ANZ) offer their own share trading that link directly to your bank account, making it easy to start investing.

You create an account, deposit some money, then choose the shares you want to buy. You'll pay a brokerage fee (usually between $5 and $20 per trade/purchase), and your shares will be stored in your account until you decide to sell them. It can all be done from your phone!

Some people trade often, while others invest and hold shares for decades. Every time you buy or sell you need to include it in your tax return, so make sure to keep records.

You can make money from shares in two main ways:

Dividends – Some companies share part of their profits with shareholders, usually once or twice a year.

Capital gains – If the share price goes up then you can sell your shares for more than you paid.

Dividends

A dividend is a payment made to shareholders from a company's profits. Companies don't have to pay dividends, but when they do it's usually:

- A fixed amount per share (e.g. \$0.30 per share), or
- A percentage of the share's current price (also called dividend yield)

Worked examples.

1. You own 500 shares in a company. They pay a dividend of \$0.40 per share.

   Total Dividend = 500 × 0.40 = \$200.

   ∴ You would receive \$200 in dividends

2. You own 800 shares in a company. The shares are currently worth \$3.00 each, and the dividend yield is 5%.

   Dividend per share = 5% of \$3.00 → 0.05 × \$3 = \$0.15

   Total Dividend = 800 × \$0.15 = \$120

   ∴ You would receive \$120 in dividends

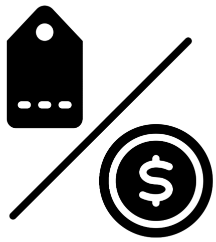
With your teacher.

1. Sienna owns 600 shares in a company that pays a dividend of \$0.35 per share. Calculate the total dividend received.

2. Taylah owns shares in three companies, as detailed below. Calculate the total dividends received.

Company	Shares Owned	Share Price	Dividend Type
Alpha Health	300	\$2.50	\$0.10 per share
Beta Energy	500	\$4.00	4% dividend yield
Gamma Retail	400	\$3.20	No dividend

PRICE-TO-EARNINGS RATIO (P/E RATIO)



The P/E Ratio helps compare how “expensive” a share is relative to its earnings.

The P/E ratio is used to:

- Help investors decide if a share is good value or overpriced
- Compare different companies, especially in the same industry
- Spot growth potential (high P/E) or steady income (lower P/E)

A **high P/E** means investors expect future growth, so they’re willing to pay more now
A **low P/E** might mean the company is undervalued or not expected to grow much.

It’s not good or bad on its own - it’s most useful when comparing companies or looking at a company’s P/E over time.

Price-to-earnings (P/E) Ratio formula:

$$P/E\ ratio\ = \frac{Share\ price}{Annual\ earnings\ per\ share}$$

Worked example.

1. A company’s shares are trading at $20.00 each. The company’s earnings per share (EPS) is $2.50. Calculate the P/E ratio.

   P/E ratio **=** $\frac{\$20.00}{\$2.50} = \$8.00$

2. Jessica is analysing two companies listed on the ASX to decide where to invest her money. She gathers the following data:

Company	Share Price	Earnings per Share (EPS)
Alpha Ltd	$36.00	$3.00
Beta Corp	$20.00	$1.00

a) Calculate the P/E ratio for each company.

Alpha Ltd	Beta Corp
$P/E\ = \frac{36}{3} = \$\mathbf{12}$	$P/E\ = \frac{20}{1} = \$\mathbf{20}$

Interpretation: The price-to-earnings ratio is $12 for Alpha Ltd and $20 for Beta Corp. This means, for Alpha Corp, investors are paying $12 for every $1 the company earns. Beta Corp would be expected to have higher growth potential.

With your teacher.

1. A company's shares are trading at \$15.00 each. The company's earnings per share (EPS) is \$5.00. Calculate the P/E ratio.

2. You are looking at two companies in the retail industry. Calculate the P/E ratio to compare the two. Which company has the lower P/E ratio and what does this suggest about the price investors are paying relative to the company's earnings?

Company	Share Price	EPS
ShopSmart	\$20.00	\$2.00
BuyBig	\$30.00	\$1.50

3. Maxiford Industrials posted annual earnings of \$1,500,000. It distributed 20% of this as a dividend to the holders of its 200,000 shares and re-invested the rest into the company. If Maxiford Industrials shares are trading at \$10.80, find:

   a) The dividend per share.

   b) The dividend yield.

   c) Divide the total earning by the number of shares to find the Earning Per Share (EPS). Use this figure to find the Price to Earnings ratio.

Try it yourself (INVESTING).

1. Match the investment to its definition:

a) Real estate	b) Pooling money to invest in a range of assets
c) Shares	d) Buying sneakers, toys, or trading cards
e) Term deposit	f) Earning interest through the bank
g) Collectables	h) Owning part of a business
i) Managed funds	j) Buying property to rent or resell

2. Calculate the total dividend amounts received: (dividend per share)

a) \$0.50 per share, 200 shares	b) \$1.20 per share, 1,000 shares
c) \$0.60 per share, 400 shares	d) \$2.50 per share, 20 shares
e) 3 cents per share, 500 shares	f) 0.25 cents per share, 5,000 shares

3. Calculate the total dividend amounts received: (percentage dividend)
   a) 1,000 shares owned, \$10 per share, 3% dividend
   b) 5% yield, \$2.50 market value per share, 500 shares
   c) \$0.13 market value per share, 10% yield, 650 shares
   d) 5,000 shares, 2.5% dividend yield, \$100 market value of share
   e) \$500 share value, 6% dividend yield, 10,000 shares owned
   f) \$0.20 share value, 1% dividend yield, 450 shares owned

4. Zara owns shares in three companies: Calculate Zara's total dividend income.

Company	Shares Owned	Share Price	Dividend Type
Alpha Ltd	300	\$3.00	\$0.10 per share
Beta Pty Ltd	500	\$4.00	5% dividend yield
Gamma Co	400	\$2.50	No dividend

5. Calculate the price-to-earnings ratio of the following:

	Company	Share price	EPS
a)	BudgetMart	\$20.00	\$2.50
b)	ValueDepot	\$18.50	\$2.00
c)	PriceWorks	\$32.20	\$4.35
d)	SmartCart	\$10.00	\$1.00
e)	BargainBase	\$14.85	\$1.75

6. Rank the companies from question 5 from highest expected growth to lowest expected growth based on their price-to-earnings ratio.

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