2026

GENERAL MATHEMATICS

Level 2

MEASUREMENT

General Mathematics: Level 2

GM2 - MEASUREMENT

By Jess Bertram

With sincere thanks to John Short and Rick Smith

ICON:	MEANING:
	Worked example
	Complete with your teacher
	Try it yourself
	Tips / shortcuts
	Investigation

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GENERAL MATHEMATICS - LEVEL 2

GM2 - MEASUREMENT

BY JESS BERTRAM

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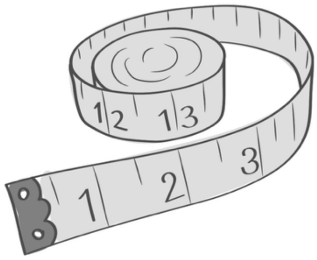
Part 1 MEASUREMENT

INTRODUCTION TO MEASUREMENT



Measurement is how we describe and compare the size of things in the real world. We use measurement every day - from checking our height, to filling up a petrol tank, to building houses and designing controversial AFL stadiums.

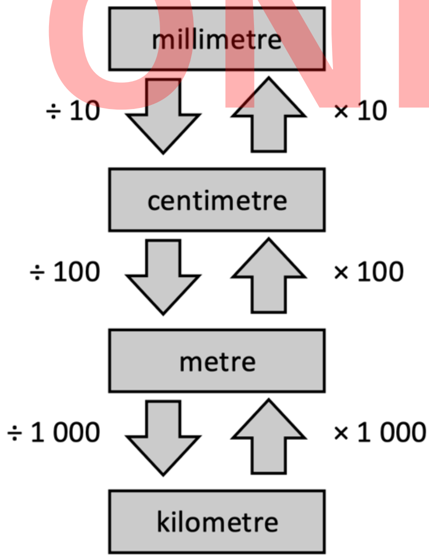
In mathematics, measurement allows us to connect numbers to real objects by using units such as centimetres, metres, and litres.

When solving problems, it's important to **use the same units** for all parts of the question. If measurements are given in different units, convert them first.

Complete with your teacher.

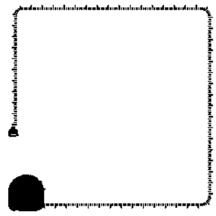
1. Convert the following measurements.

Original	Convert to
100mm	cm
3,506mm	cm
200cm	m
1,346cm	m
2,000m	km
3,506mm	m
210km	m
35m	cm
22.5cm	mm
2.5km	mm



2. Add together the following: 125mm + 35.6cm +1050mm + 1.5m + 27.4cm

PERIMETER



Perimeter is the distance around the outside of a two-dimensional shape. It is a linear measurement, usually measured in millimetres (mm), centimetres (cm), metres (m) or kilometres (km).

To calculate perimeter, we add together the lengths of all the sides of the shape.

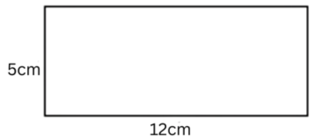
In this section we will cover how to find the perimeter of the following shapes:

- Rectangles
- Triangles
- Parallelograms
- Circles (including sectors)

Worked Examples. (rectangles, triangles & parallelograms)

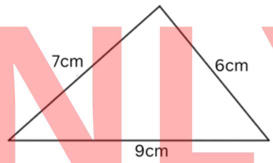
1. A rectangle has a length of 12 cm and a width of 5 cm.

   Perimeter = 5 + 5 + 12 + 12 = ***34 cm.***



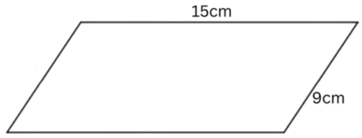
2. A triangle has sides of 6 cm, 7 cm, and 9 cm.

   Perimeter = 6 + 7 + 9 = ***22 cm****.*



3. A parallelogram has a base of 15 cm and a side length of 9 cm.

   Perimeter = 15 + 9 + 15 + 9 = ***48 cm****.*



With your teacher. Find the perimeter of the following shapes.

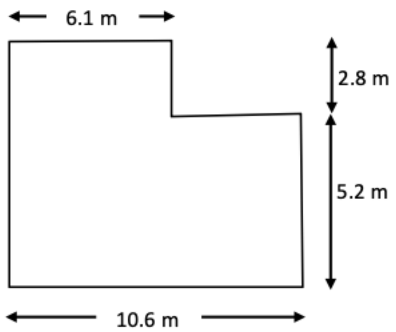
1. Rectangle with length 8 cm and width 3 cm.

2. Triangle with side lengths 5 cm, 12 cm, and 13 cm.

With your teacher.

1. A rectangle has a perimeter of 50 cm and a width of 10 cm. What is its length?

2. Find the perimeter of the following shape:



Try it yourself (PERIMETER): **Answers page 56**

1. Change the followings units. Round to 2 decimal places (2dp) if needed.

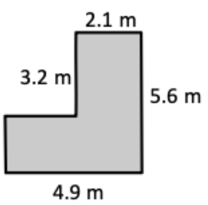
a. 5m in cm	b. 4.63m in cm	c. 5.4m in mm	d. 5.28m in mm
e. 5276m in km	f. 756m in km	g. 1235mm in cm	h. 135mm in cm
i. 5682mm in m	j. 3682mm in m	k. 39cm in mm	l. 820cm in mm
m. 350cm in m	n. 1274cm in m	o. 3.5km in m	p. 0.85km in m

2. Arrange each of the following in ascending order (smallest to biggest):

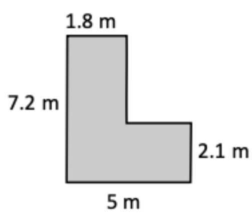
   1m, 3.5cm, 1cm, 450mm, 25mm, 2370mm, 320cm, 2.2m, 5mm

3. Find the perimeter of the following shapes.

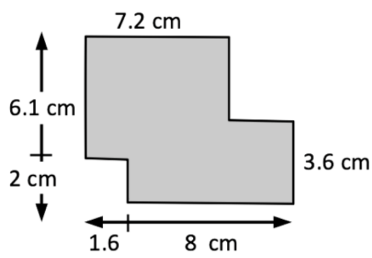
a.



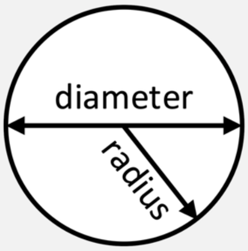
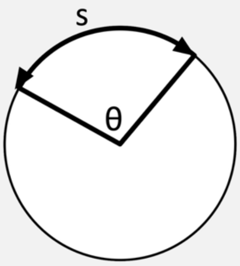
b.



c.



4. A triangle has two equal sides of 8cm and a base of 6cm. Find its perimeter.
5. A builder is constructing a deck that is 3.2m wide. How many decking boards will be used if each decking board is 70mm wide and a 6mm gap is to be left between each board? Assume no gap at the end.
6. A carpet costs $140 per metre length and is cut from a roll that is 4m wide. Find the lowest cost of carpeting a hall which measures 11500mm x 8400mm.

Perimeter with circles (circumference)		
	Full circumference $C = \pi d$ or $C = 2\pi r$	Where: d = diameter r = radius
	Arc length $S = \pi d \times \frac{\theta}{360}$ or $S = 2\pi r \times \frac{\theta}{360}$	Perimeter of a sector $P_s = S + 2r$

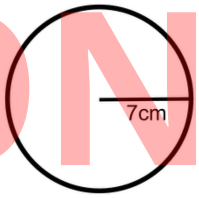
Worked examples.

1. Find the circumference of a circle with a radius of 7cm.

$C = 2\pi r$

$C = 2 \times \pi \times 7$

$C = 43.98cm$

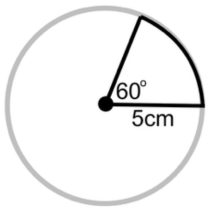


2. Find the perimeter of a sector that has radius of 5cm and an included angle of 60°.

$P_s = \left[2\pi r \times \frac{\theta}{360}\right] + 2r$

$P_s = \left[2 \times \pi \times 5 \times \frac{60}{360}\right] + 2 \times 5$

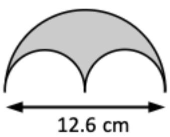
$P_s = 15.24cm$



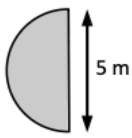
With your teacher.

Find the perimeter of the following shapes.

a.

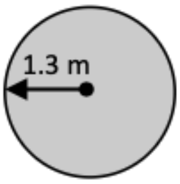


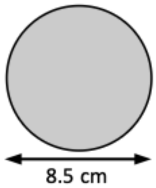
b.

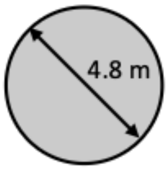


Try it yourself (CIRCUMFERENCE): **Ans pg56**

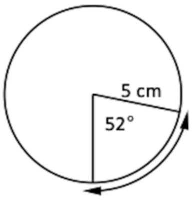
1. Find the circumference of the following circles.

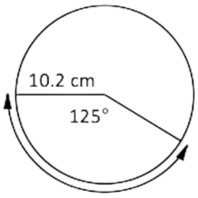
a. 

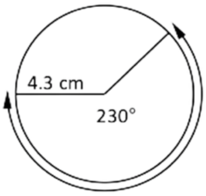
b. 

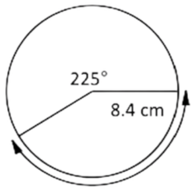
c. 

2. Find the circumference of a circle with diameter 12 cm.
3. Find the arc length of the following:

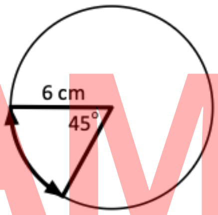
a. 

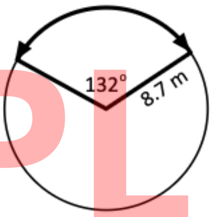
b. 

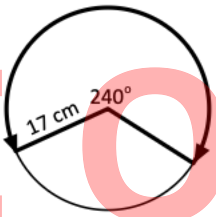
c. 

d. 

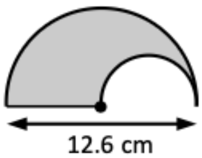
4. Find the perimeter of the following sectors.

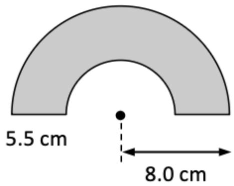
a. 

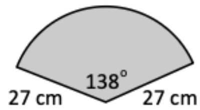
b. 

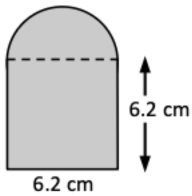
c. 

5. Find the perimeter of the following shapes.

a. 

b. 

c. 

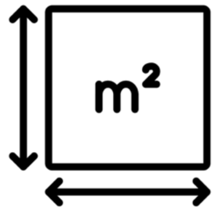
d. 

6. A windscreen wiper has a length of 42cm and swings through an arc of 125°. Find the distance moved by the tip of the blade with each swing. (arc length)
7. A wrist-watch has a diameter of 3cm. How far does the tip of the second hand move in 5 seconds?
8. A pendulum swings through an angle of 15°. If the string is 1.2m how far does the pendulum bob move with each complete swing (back and forwards)?
9. A child is on a swing which has ropes of length 3m, moving through an angle of 35°. How far has the child travelled after 1 minute if a complete swing (back and forwards) is made every 4 seconds?

AREA

Area is the amount of space inside a two-dimensional shape.

It is measured in square units such as mm^2, cm^2, m^2 and km^2.



Make sure all dimensions are in the same unit before you start.

Area of a rectangle

$$\boldsymbol{A = L \times W}$$

L = Length of shape

W = Width of shape

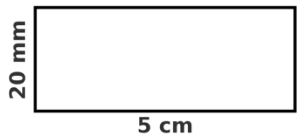
Worked example.

Find the area of a rectangle with sides of 20mm and 5cm.

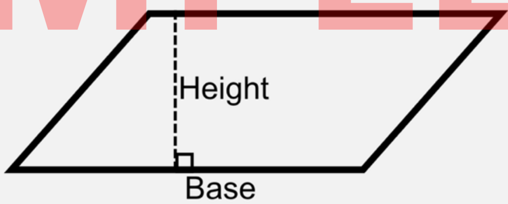
L = 5cm, W = 2cm ← *Convert the units **first***

$A = 5 \times 2$

$A = 10cm^2$



Area of a parallelogram



$$\boldsymbol{A = b \times h}$$

Where:

b = base

h = height

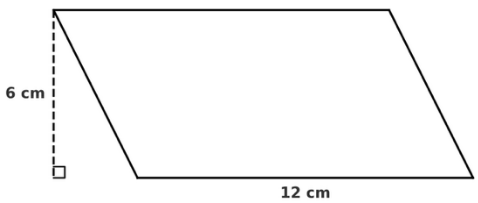
⚠ Important: The **height** is the perpendicular distance from the base to the opposite side — not the slanted side length.

Find the area of a parallelogram with a base of 12 cm and a height of 6 cm.

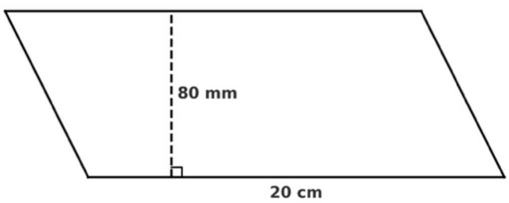
$A = b \times h$

$A = 12 \times 6$

$A = 72cm^2$



With your teacher. Find the area of the following shapes.



Try it yourself (AREA OF RECTANGLES AND PARALLELOGRAMS): **Ans pg56**

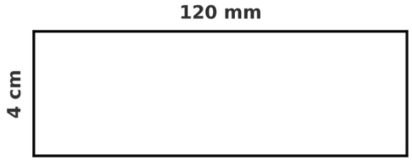
Ans pg56

Find the area of the following shapes.

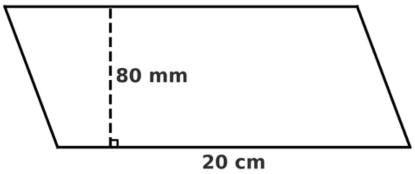
1. Rectangles with sides of:
   a. 8cm and 3cm
   b. 10mm, 5cm, 10mm and 5cm

2. Parallelograms with dimensions:
   a. h = 10mm, b = 15mm
   b. h = 100cm, b = 2m

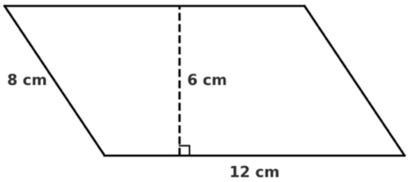
3.



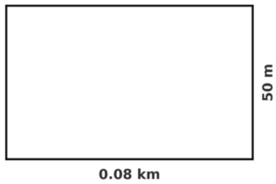
4.

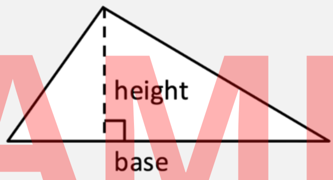
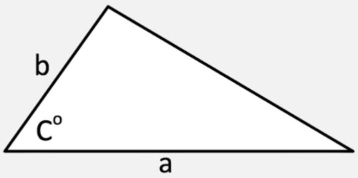
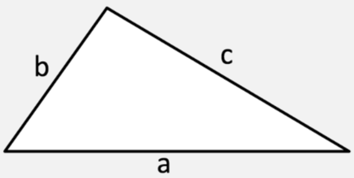


5.



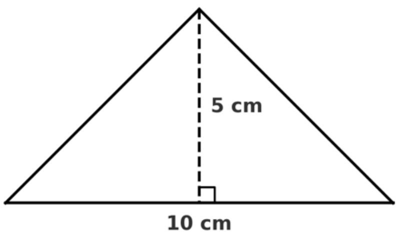
6.



Area of triangles		
	$\boldsymbol{A = \frac{1}{2} \times b \times h}$	b = base h = height (base and height must meet at a 90° angle)
	$\boldsymbol{A = \frac{1}{2} ab \times \sin C}$	Angle C is the included angle between sides a and b.
	$\boldsymbol{A = \sqrt{s(s-a)(s-b)(s-c)}}$ $\boldsymbol{s = (a+b+c) \div 2}$	Heron's formula

Worked examples. Use an appropriate formula to find the areas of the triangles.

1.



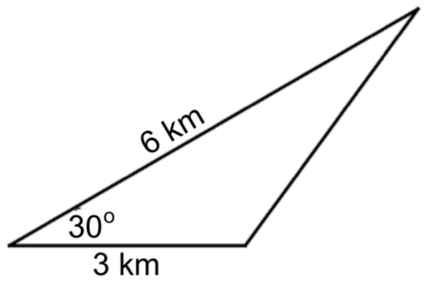
$$A = \frac{1}{2} \times b \times h$$

$$A = \frac{1}{2} \times 10 \times 5$$

$$A = 25cm^2$$

Worked examples (continued)

2.



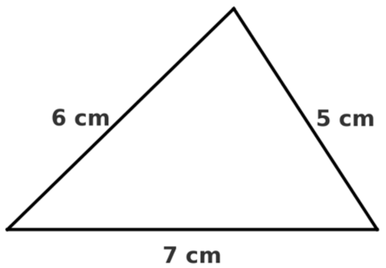
$$A = \frac{1}{2}ab \times \sin C$$

$$A = \frac{1}{2} \times 3 \times 6 \times \sin 30$$

$$A = 4.5km^2$$

Make sure your calculator is in degrees mode for this course

3.



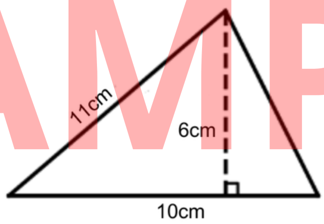
$$s = (6 + 5 + 7) \div 2 = 9$$

$$A = \sqrt{9(9-6)(9-5)(9-7)}$$

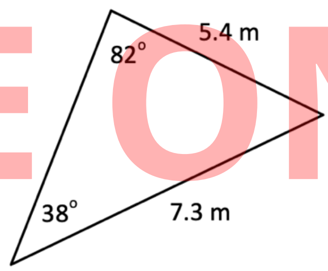
$$A = 14.70km^2$$

With your teacher. Use an appropriate formula to find the area of each triangle.

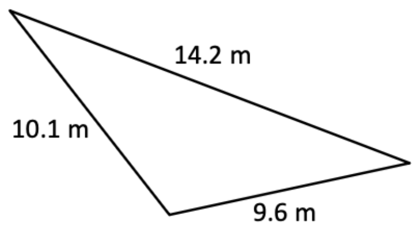
1.



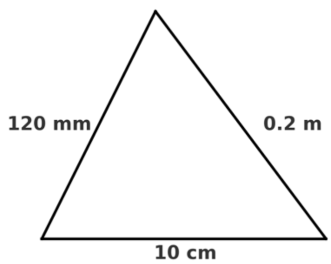
2.



3.

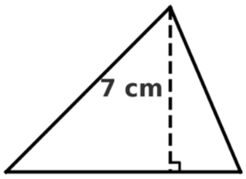


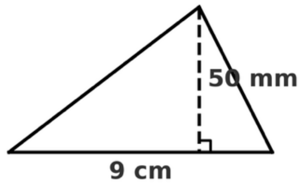
4.

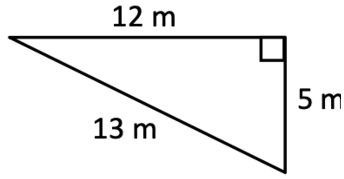


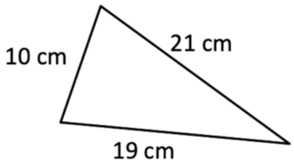
Try it yourself (AREA OF TRIANGLES): **Ans pg56**

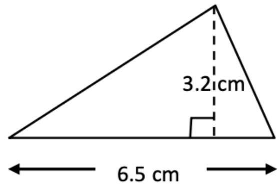
1. Use an appropriate formula to find the area of each of the following triangles.

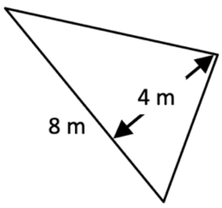
a.


b.


c.


d.


e.


f.


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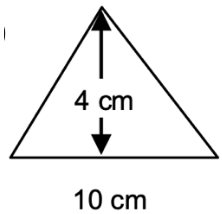

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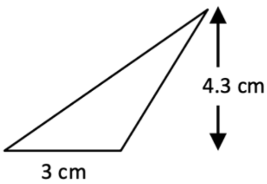

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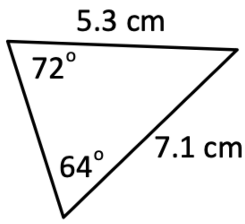

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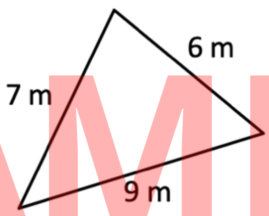

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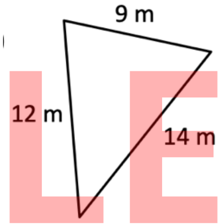

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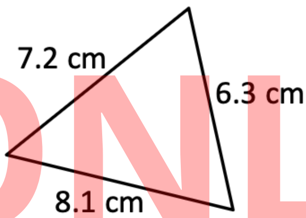

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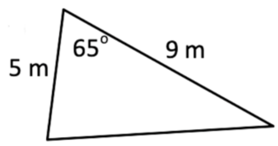

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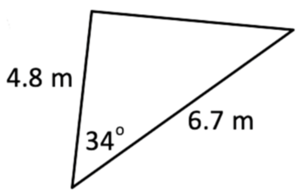

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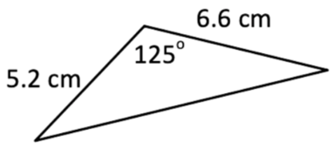
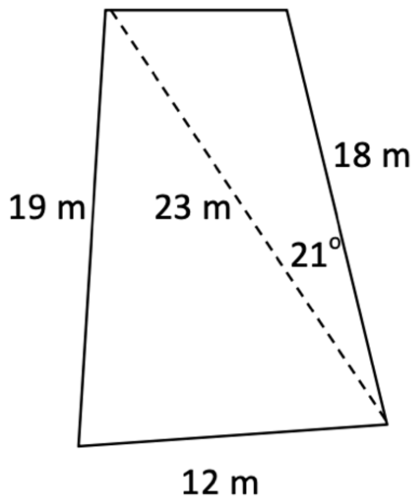
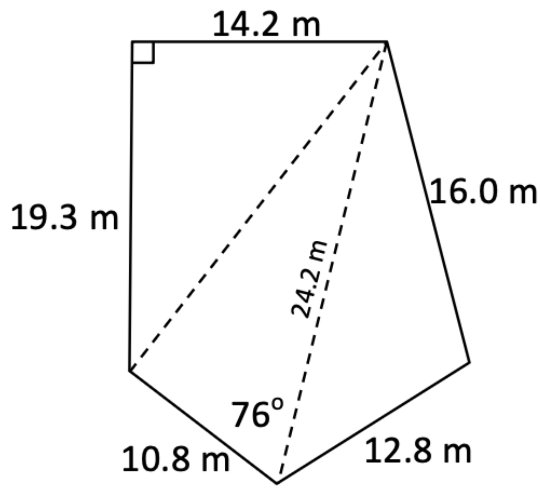
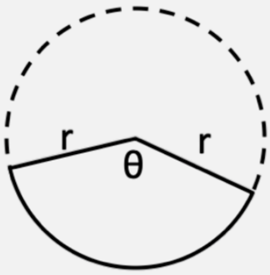


2. Find the area of each of the following shapes.

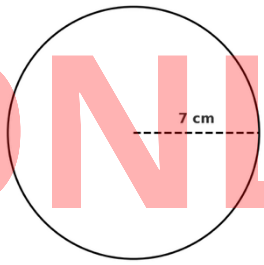
a.


b.


Area of circles		
	$A = \pi r^2$	Area of a whole circle. r = radius
	$A = \pi r^2 \times \frac{\theta}{360}$	Area of a sector (part circle)

Worked examples.

1. Find the area of a circle with a radius of 7cm.

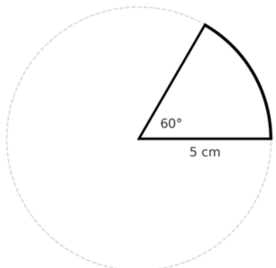


$A = \pi r^2$

$A = \pi \times 7^2$

$A = 153.94cm^2$

2. Find the area of a 60° sector with a radius of 5cm.



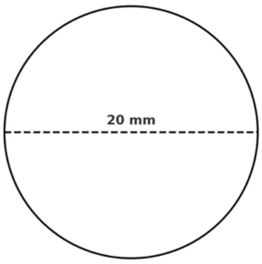
$A = \pi r^2 \times \frac{\theta}{360}$

$A = \pi \times 5^2 \times \frac{60}{360}$

$A = 13.09cm^2$

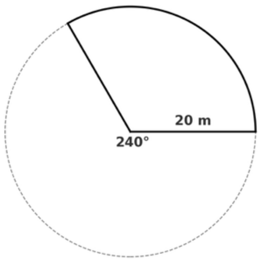
With your teacher.

1. Find the area of this circle.



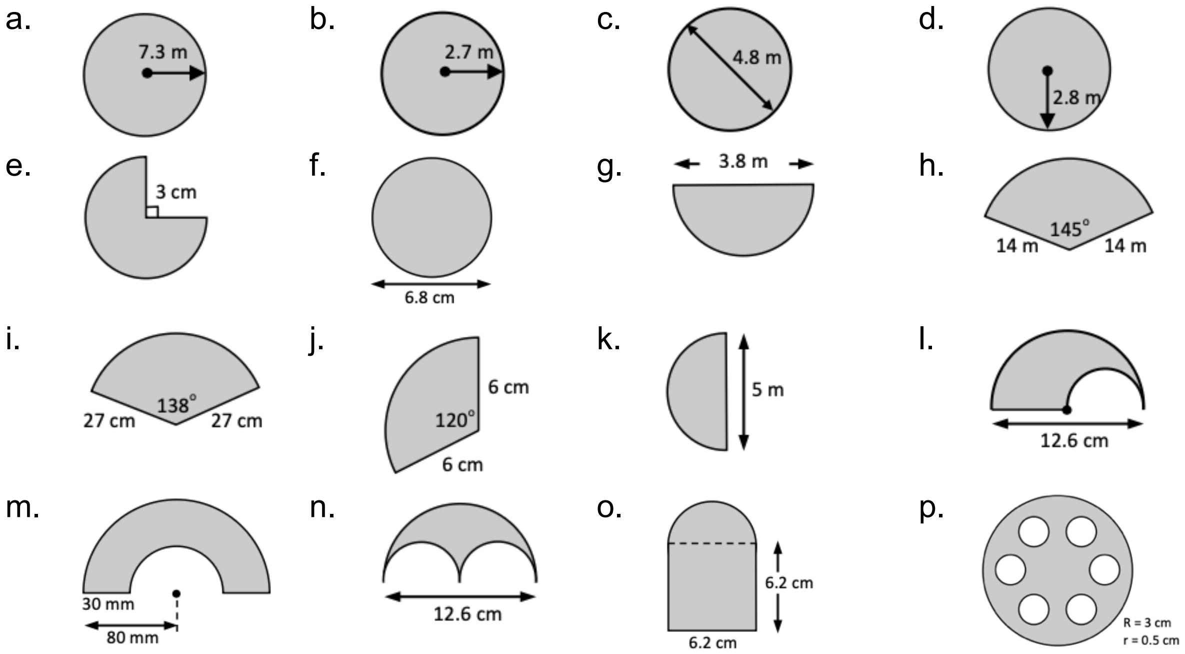
2. Find the area of this sector.

Be careful with the angle, the 240 ° is outside the sector.

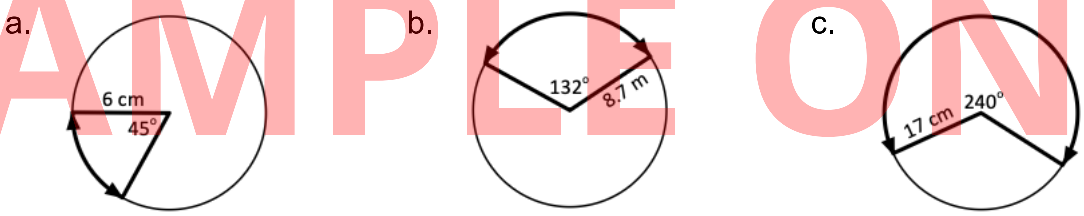


Try it yourself (AREA OF CIRCLES): **Ans pg56**

1. Find the area of the shaded region in each of the following shapes. (2.d.p)



2. Find the area of each of the below sectors.

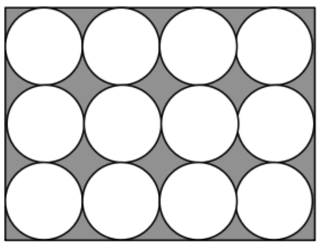


3. A landscape gardener wants to fill a circular patio area with decorative gravel. The patio area has a diameter of 3.5m. If the gravel is going to cost $20 per m^2 how much will it cost to fill the patio area?

4. A groundsman is asked to sew new grass seeds to a circular cricket ground 60m in radius. How much seed is required if the recommended coverage is 50g per square metre?

5. An archery target has a ‘face’ (outer ring) with a diameter of 1.5m. The diameter of the inner ‘bulls-eye’ is 0.4m. Express the area of the bulls-eye as a percentage of the total target area.

6. Circular tin lids are cut from a sheet of tin plate as shown in the diagram. Calculate the percentage of the tin sheet that will be wasted.



Area of composite shapes

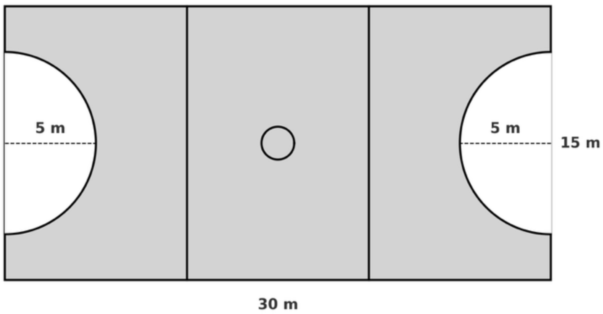
Composite shapes are made from two or more standard shapes joined together.

To find the area:

1. Split the composite shape into smaller standard shapes
2. Find the area of each smaller shape
3. Add or subtract areas to construct the composite shape

Worked Example:

Liz Watson accidentally stepped in paint before starting a game of netball. She plays the position of centre the whole game, getting paint all over the court except for in the shooting semi-circles at each end.



a. Find the impacted area of the netball court. *(it has been shaded in the above diagram)*

Composite shape is made up of a rectangle minus two semi-circles.

$Area_1$ **= rectangle.**

L = 30m, W = 15m

$$A_1 = L \times W$$

$$A_1 = 30 \times 15$$

$$A_1 = 450\text{m}^2$$

$Area_2$ **= semicircle.**

r = 5m

$$A_2 = \pi r^2 \div 2$$

$$A_2 = \pi \times 5^2 \div 2$$

$$A_2 = 39.27\text{m}^2$$

Total area

$$A = A_1 - A_2 - A_2$$

$$A = 450 - 39.27 - 39.27$$

$$A = 371.46\text{m}^2$$

Total area of the court impacted is 371.46m^2

b. The netball team is made to pay for the cost of paint to repaint the impacted areas of the court. If paint is \$50 a litre (sold only in whole litres), and one litre covers 10m^2, how much will it cost to repaint the impacted areas of the court?

$371.46m^2 \div 10$ = 37.15 litres → They will need 38 litres of paint.

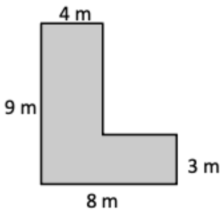
38 × \$50 = \$1,900

It will cost the team \$1,900 to pay for the paint.

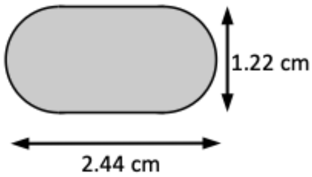
Try it yourself (COMPOSITE AREA): **Ans pg56**

1. Find the shaded area of the following shapes.

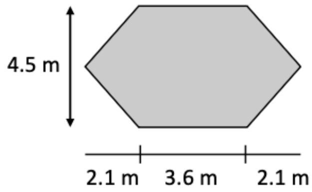
a.



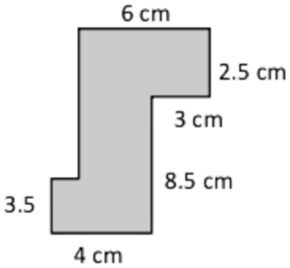
b.



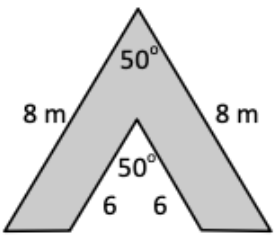
c.



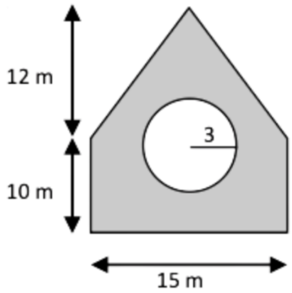
d.



e.



f.



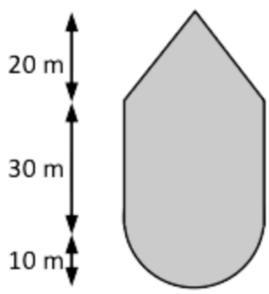
2. A patio measuring 6.2m x 3m is to be paved with square tiles, each 0.6m x 0.6m
   a. Find the number of tiles required by comparing areas
   b. Find the number of tiles required by considering the number of rows and columns of tiles needed.
   c. Why are the answers different? Which is correct?

3. A rectangular garden bed measures 6 m by 4 m. A semicircular garden with diameter 4 m is added to one of the short sides. Find the total area of the garden bed and the path.

4. A sandpit is shaped like a square of side 5 m with a circular hole in the middle for a climbing pole (radius 0.5 m). Find the area of the sandpit's surface.

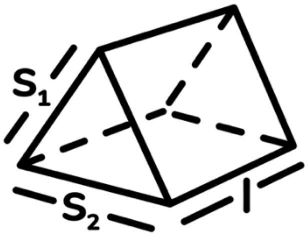
5. The roof of a shed is shaped like an isosceles triangle with base 12 m and height 4 m, sitting on top of a rectangular wall 12 m wide and 8 m tall. Find the total area of the front face of the shed.

6. Abby wants to make an arena for her horses. She wants one end with curved sides so she can lunge her horse on a lead line. She wants the other end to be pointed so she can corner and catch any naughty horses. She designs the arena to the right. Calculate the area.



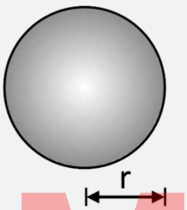
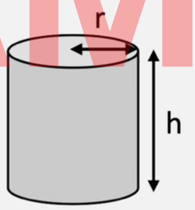
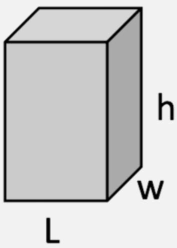
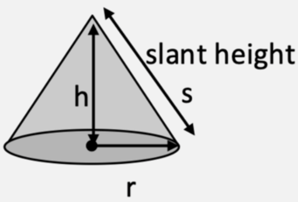
TOTAL SURFACE AREA (TSA)

Total surface area is the total area of all the outside surfaces of a 3D object.



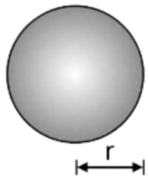
Imagine you were painting a garden shed. The total surface area of the shed would be the area of the roof + all the areas of all the external walls. Finding the total surface area would help you work out how much paint you would need.

Surface area is always measured in square units (such as cm² or m²)

Total surface area (TSA) formula		
	Sphere	$TSA = 4\pi r^2$
	Cylinder	$TSA = 2\pi rh + 2\pi r^2$
	Rectangular prism	$TSA = 2LW + 2LH + 2WH$
	Cone	$TSA = \pi rs + \pi r^2$

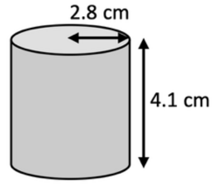
Worked examples.

1. Calculate the total surface area of a sphere with a 5cm radius.



$$TSA = 4\pi r^2$$
$$TSA = 4 \times \pi \times 5^2$$
$$TSA = 314.16cm^2$$

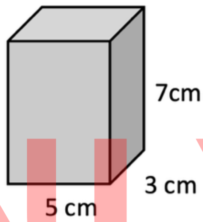
2. Find the TSA of a cylinder with a radius of 2.8cm and a height of 4.1cm.



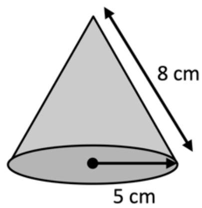
$$TSA = 2\pi rh + 2\pi r^2$$
$$TSA = 2 \times \pi \times 2.8 \times 4.1 + 2 \times \pi \times 2.8^2$$
$$TSA = 121.39cm^2$$

With your teacher.

1. Find the TSA of a rectangular prism with sides of 7cm, 3cm and 5cm.



2. Find the TSA of a cone with a radius of 5cm and a slant height of 8cm.



Finding Total Surface Area (TSA) without a formula

Not all 3D shapes have a specific formula for finding TSA.

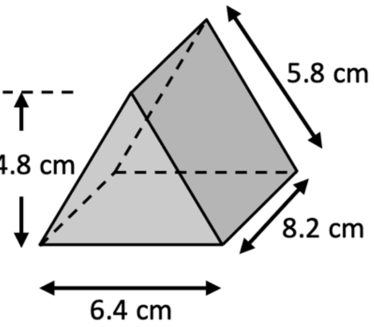
Shapes such as pyramids, prisms, and most composite shapes require a different approach.

1. Break each surface up into small, standard shapes (called ‘faces’)
2. Calculate the area of each face
3. Add all the faces together

Worked example – TSA without a formula.

Break the 3D shape up into separate surfaces (faces).

This shape has 2 triangle faces and 3 rectangle faces.



A_1 = 2 x triangles (height of 4.8cm, base of 6.4cm)

A_2 = Base rectangle (length of 8.2cm, width of 6.4cm)

A_3 = 2 x side rectangles (length of 8.2cm, width of 5.8cm)

A_1 triangles

$A_1 = \frac{1}{2}bh$

$A_1 = \frac{1}{2} \times 6.4 \times 4.8$

$A_1 = 15.36cm^2$

A_2 base rectangle

$A_2 = L \times W$

$A_2 = 8.2 \times 6.4$

$A_2 = 52.48cm^2$

A_3 side rectangles

$A_3 = L \times W$

$A_3 = 8.2 \times 5.8$

$A_3 = 47.56cm^2$

The total surface area would be:

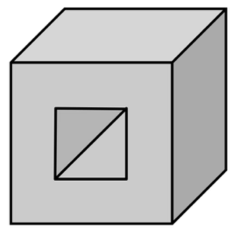
$TSA = A_1 + A_1 + A_2 + A_3 + A_3$

$TSA = 15.36 + 15.36 + 52.48 + 47.56 + 47.56$

$TSA = 178.32cm^2$

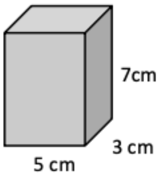
With your teacher.

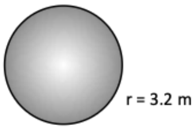
A cube with side lengths of 10cm has a square hole cut out of it. The square hole has sides of 5cm. Find the total surface area of the shape (including inside the hole!)

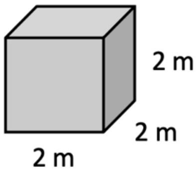


Try it yourself (TOTAL SURFACE AREA): **Ans pg56**

1. Find the total surface area (TSA) for the following shapes.

a.


b.


c.


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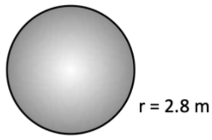

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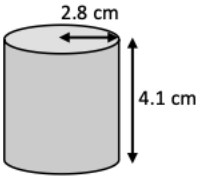

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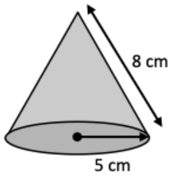

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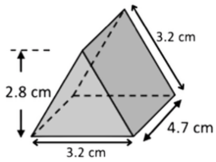

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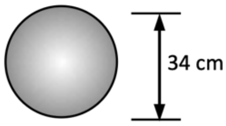
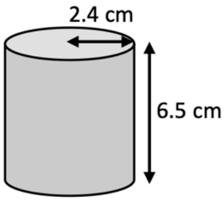
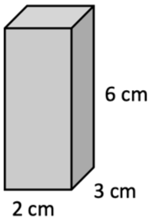
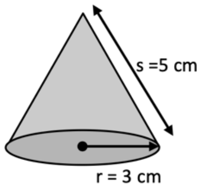
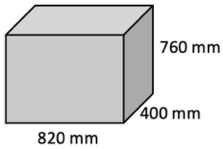


i.


j.


k.


l.


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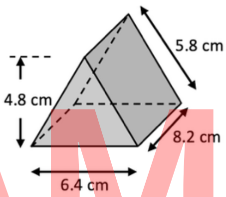

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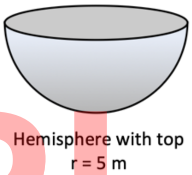

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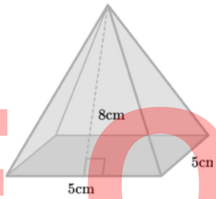

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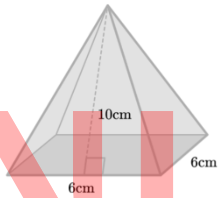
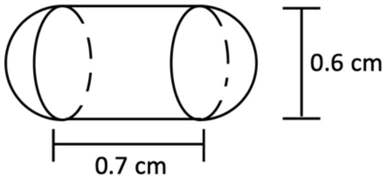


2. Find the area of land on Earth's surface given that the Earth has a radius of 6371km and 71% of its surface is ocean. (answer to the nearest million km^2).

3. A cyclist is building a wood box so that he can transport his bicycle interstate via plane. The unassembled bike can fit into a box which measures 1.2m x 0.75m x 0.35m. The bike itself weighs 9.5kg and plywood weighs 2.65kg per m^2. Will the box be allowed if the airline has a limit of 20kg on all air freight?

4. Gelatine capsules are often used in the preparation of pharmaceuticals as they are easy to swallow and are tasteless. Find the surface area of the capsule in the diagram.



5. Rainwater tanks are constructed from galvanised iron. Thin sheets which cost \$6.75 per m^2 are needed for the top and the bottom of the tank. The sides need more substantial iron which costs \$12.40 per m^2. Find the cost of manufacturing a cylindric tank with a radius of 1.2m and height of 1.8m.

VOLUME

Volume is the amount of space inside a 3-D object. You can think of it as how much liquid something can hold.

We measure volume in cubic units (cm³, m³, etc.), or in litres (L, mL etc) if talking about liquids.

In this workbook you will calculate the volume of:

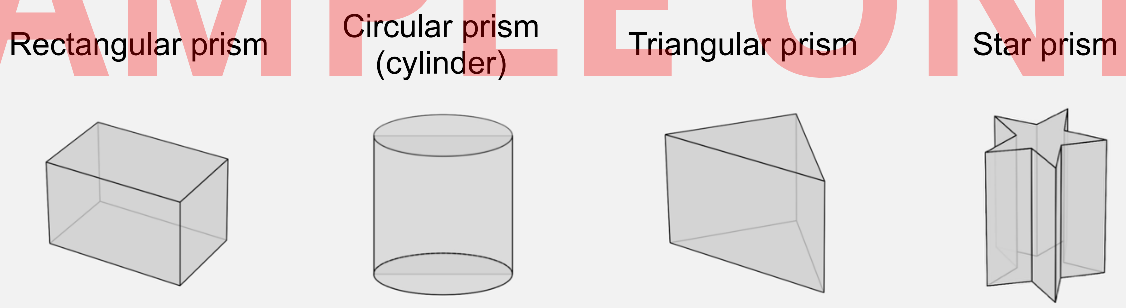
- Prisms (including rectangular prisms and cylinders)
- Spheres
- Cones
- Pyramids
- Composite shapes

Volume of a prism

A prism is a 3D shape.

It has the same 2D shape on each end (base shape), connected by a 'tunnel'

Some examples of prisms are:



To find the volume of a prism use the formula:

$$\boldsymbol{V = Ah}$$

Where:

A = Area of the base shape

h = height (distance from one base shape to the other)

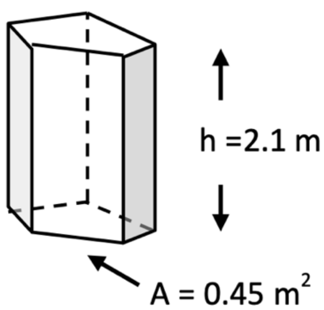
Worked example.

Find the volume of the prism in the diagram.

$V = Ah$

$V = 0.45 \times 2.1$ ← *The area of the base is given in this shape*

$V = 0.945m^3$



Worked example.

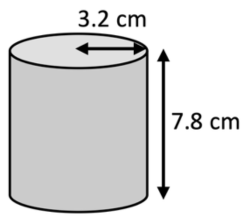
Find the volume of a cylinder with a radius of 3.2cm and a height of 7.8cm

$V = Ah$ ← "A" represents the area of the base shape.

$V = (\pi r^2) \times h$ ← Replace the "A" with the formula for area of a circle.

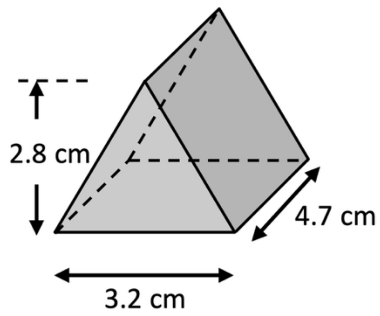
$V = (\pi \times 3.2^2) \times 7.8$

$V = 250.93cm^3$

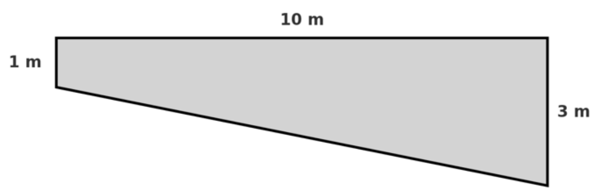


With your teacher.

1. Find the volume of the triangular prism in the diagram.

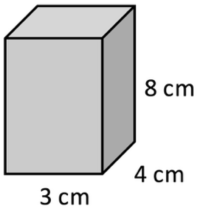


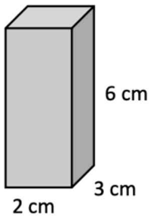
2. Ariarne Titmus has used her Olympic prize money to install a pool in her backyard. The pool is 10 metres long and 4 metres wide. The deep end is 3m deep, and the shallow end is 1m deep.
   Calculate the volume of water needed to fill the swimming pool $(1m^3 = 1{,}000L)$

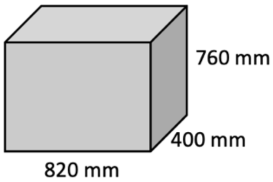


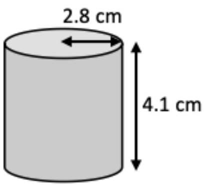
Try it yourself (VOLUME OF PRISMS): **Ans pg56**

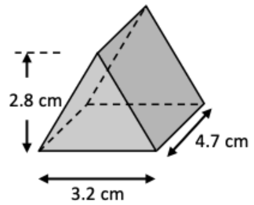
1. Find the volume of the following prisms (give your answers in cm^3)

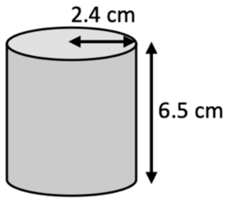
a.


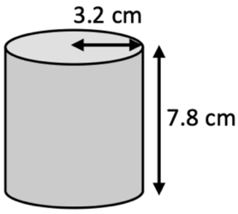
b.


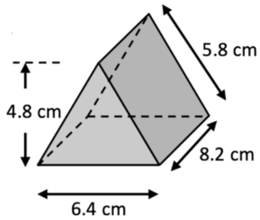
c.


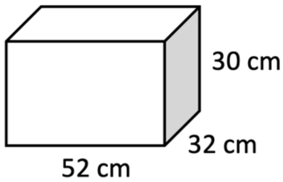
d.


e.


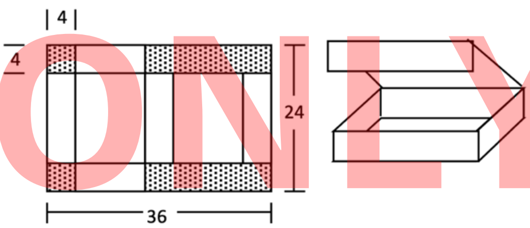
f.


g.


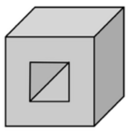
h.


i.


2. Charlotte wants to make cardboard boxes for her catering business. She makes each box out of a sheet of cardboard measuring 36cm x 24cm. The box has a lid that folds over and tucks in. Charlotte wants to make one giant slab of fudge that fits perfectly in the box. Find the volume of the box.

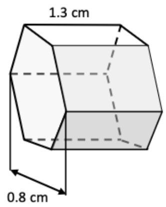


3. A cube of sides 4cm has a square hole of sides 2cm cut through it. Find the total volume of the solid.



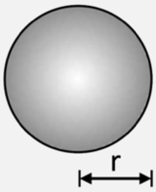
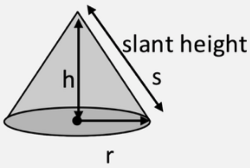
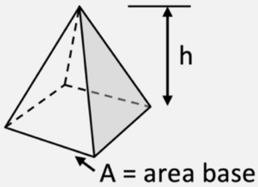
4. A shipping company charges $25 plus $8.50 per cubic metre of volume. How much would it cost to ship a crate which measures 2.5m x 1.4m x 0.9m?

5. Bees store honey in a comb which they construct in the shape of a hexagonal prism to the dimensions shown. Find the amount of honey that can be stored in each honeycomb cell. *Hint: Divide the hexagon into 6 equilateral triangles and find the area using* $A = \frac{1}{2}ab \times \sin C$



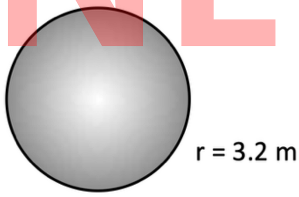
6. How many boxes measuring 24cm x 50cm x 15cm can be packed into a carton that measures 48cm x 100cm x 90cm?

7. Find the height of a prism if its volume is $45m^3$ and has a base area of $16m^2$.

Volume formula Some 3D shapes have specific formulas for calculating the volume.		
	Sphere	$V = \frac{4}{3}\pi r^3$
	Cone	$V = \frac{1}{3}\pi r^2 h$
	Pyramid	$V = \frac{1}{3}Ah$

Worked example.

Find the volume of a sphere with a radius of 3.2 metres.



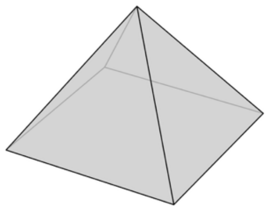
$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3} \times \pi \times 3.2^3$$

$$V = 137.26m^3$$

With your teacher.

Billy is making a model set of pyramids for his school project on Egypt. He has made a mould for the pyramid with a square base of 20cm x 20cm, and a height of 30cm. He wants to build the pyramids out of plaster. Plaster is sold in 500mL bags. How many bags would he need? $(1cm^3 = 1mL)$



Volume of composite shapes

When calculating the volume of composite shapes

1. Break into smaller familiar / standard shapes
2. Calculate volume of smaller shapes
3. Add or subtract shapes to make the composite shape.

Worked example.

Students in Mrs Thorp's science class are learning about rainfall.

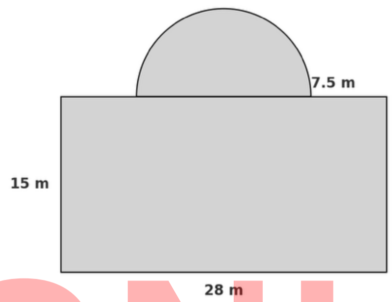
On a particularly rainy day, they measure the rainfall on the school basketball court and attached seating area.

The rectangular basketball court measures 28m x 15m.

The viewing area is a semi-circle with a radius of 7.5m.

The students measure 20mm of rain evenly over the entire area.

Find the total amount of rainfall.



1. Find the volume of both shapes.

V_1 = volume of rectangular prism

V_2 = volume of semi-circle prism

$V_1 = Ah$

$V_1 = (L \times W) \times h$

$V_1 = (15 \times 28) \times 0.02$ ← Convert the 20mm to 0.02m

$\boldsymbol{V_1 = 8.4m^3}$

$V_2 = Ah$

$V_2 = (\frac{1}{2}\pi r^2) \times h$

$V_2 = (\frac{1}{2} \times \pi \times 7.5^2) \times 0.02$

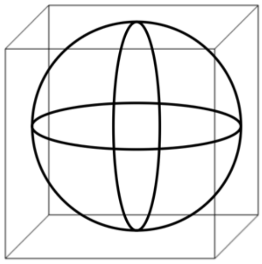
$\boldsymbol{V_2 = 1.77m^3}$

$\boldsymbol{V_1 + V_2 = 10.17m^3}$

Total amount of rainfall is $10.17m^3$ or 10,170 litres of rain $(1m^3 = 1{,}000L)$

With your teacher.

1. A basketball signed by Michael Jordan is being shipped to Australia. The ball has a diameter of 25cm. The ball is packed tightly into a box and the remaining space in the box is filled with packing peanuts. What volume of the box would need to be filled with packing peanuts?

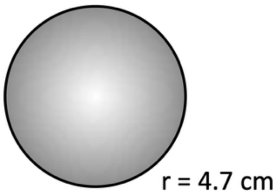


2. Billy is still working on his model set for his Egypt school project. He decided that his plaster pyramid was too simple, he now wants to make it fancier. He decides he will make 3 pyramids and mount them on a rectangular block; all made of plaster. The pyramids still have a square base of 20cm x 20cm, and a height of 30cm. The rectangular base will have a surface area of 1m x 1m and a height of 20cm. Calculate the volume of plaster needed to make his vision.

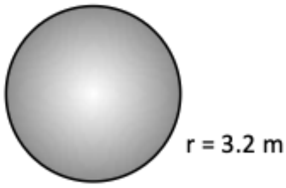
Try it yourself (VOLUME): **Ans pg57**

1. Find the volume of each shape.

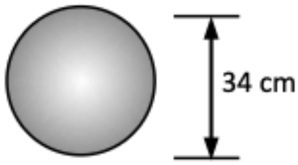
a.



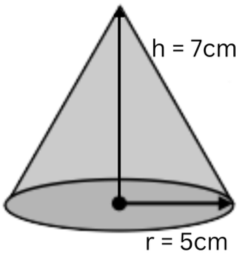
b.



c.



d.



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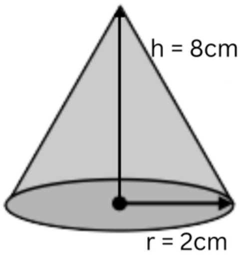

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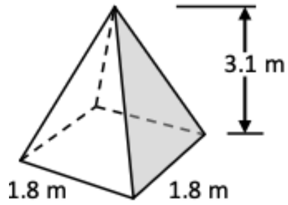
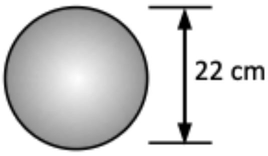


g.



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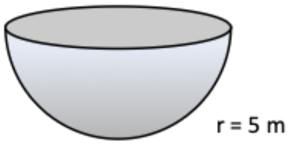
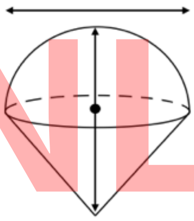
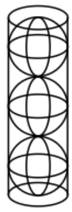


2. A hot air balloon has a diameter of 30m and an overall height of 35m. Its shape can be considered a hemisphere on top of a cone (as shown in the diagram). Find the total volume of the balloon.



3. A cylindrical tin is designed to hold 3 tennis balls, each with radius 3.5 cm. The tin fits tightly around the balls. After the container has been filled, the extra air is extracted to ensure the balls retain their bounce. Find the volume of air that must be extracted.



4. A rectangular block of play-dough with dimensions 5cm x 8cm x 4cm is rolled into a ball. Assuming no play-dough is lost, what is the radius of the ball?

5. A candle is produced in the shape of a square based pyramid. Its base sides are 6cm and has a vertical height of 10cm. How long with the candle last if each cm^3 of wax will burn for 18 minutes?

6. A block of butter measuring 5.2cm x 5.2cm x 11cm is melted into a round saucepan (cylinder) with a diameter of 12.6cm. To what depth is the saucepan filled?

Part 3 TRIGONOMETRY

Trigonometry is the branch of mathematics that deals with triangles. The word comes from “trigon” (triangle) and “metron” (measure) in greek, and translates to “measuring triangles”.

Trigonometry allows us to solve real-life problems like finding heights of buildings, lengths of ramps, and distances across rivers. It is frequently used in navigation, construction, and engineering.

In this trigonometry unit we will cover:

- Using Pythagoras’ theorem
- Using trigonometric ratios (SOH CAH TOA)
- Finding the area of a triangle
- Using angles of elevation and depression

PYTHAGORAS’ THEOREM

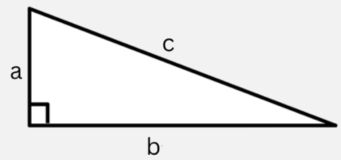
Pythagoras of Samos was a very interesting dude who live around the time of 500BC. He was a Greek philosopher, mathematician, and even started his own cult that believed numbers had mystical powers and beans contained the souls of the dead. He did lots of stuff, but his main claim to fame is Pythagoras’ theorem, which links the three sides of a right-angled triangle together.

Pythagoras’ theorem

$$a^2 + b^2 = c^2$$

$$c^2 - b^2 = a^2$$

$$c^2 - a^2 = b^2$$

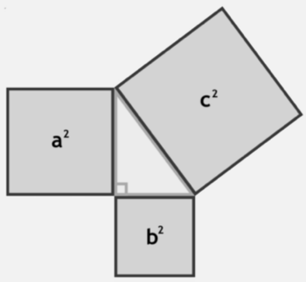


Where:

a & b = the two shorter sides of the triangle

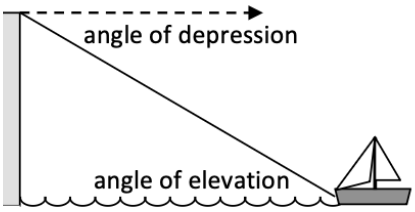
c = the hypotenuse (longest side, opposite the right angle)

It states: in a right-angled triangle, the square of the hypotenuse is equal to the square of the two other sides added together.



ELEVATION AND DEPRESSION

We often use trigonometry to solve problems about looking up or down at objects. This is known as elevation and depression.



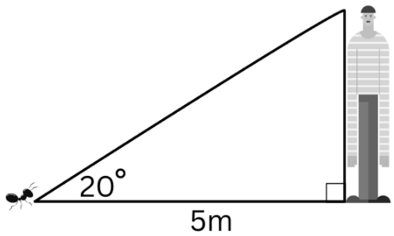
Angle of Elevation: Looking **up** from a horizontal.

Angle of Depression: Looking **down** from a horizontal.

Both angles are measured from a **horizontal line** (not from the object itself).

Worked examples.

1. An ant on the ground is looking up at the tallest man he has ever seen. The ant is 5 metres away from the man and is looking up at an angle of of 20°. How tall is the man?



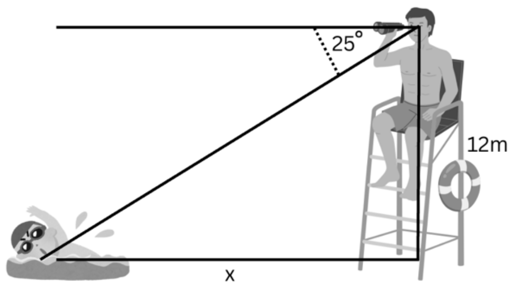
$$\tan(20°) = \frac{x}{5}$$

$$x = 5 \times \tan(20°)$$

$$x = 1.82m$$

The man is 1.82 metres tall

2. A lifeguard is sitting at the top of a lookout tower that is 12 m high. He spots a swimmer in the water at an angle of depression of 25°. How far is the swimmer from the base of the tower?



90° – 25° = 65°

$$\tan(65°) = \frac{x}{12}$$

$$x = 12 \times \tan(65°)$$

$$x = 25.73m$$

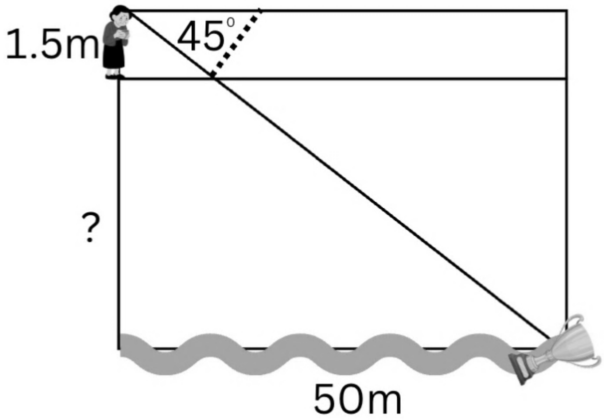
The swimmer is 25.73 metres from the base of the tower.

To use the trig ratios you need the angle **inside** a triangle.

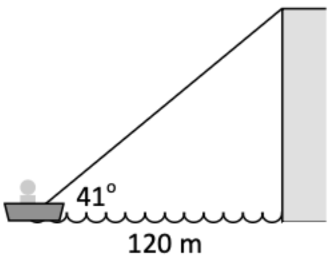
Angles of depression often require an extra step to find the interior angle.

With your teacher.

1. Elizabeth Blackburn dropped her Nobel Prize trophy off a cliff near her childhood home in Launceston. Elizabeth contemplates jumping in after it but is not sure how high the cliff is. The trophy is currently floating 50m away from the base of the cliff, and Elizabeth is looking down at it at an angle of depression of 45° . If Elizabeth is 1.5 metres tall, how tall is the cliff?



2. A small boat 120m out to sea notes that the angle of elevation to of the top of a cliff is 41°. Find the height of the cliff.



Worked example.

A woman standing at position A finds that the angle of elevation to the top of a 220m high tower is 37°. Upon walking to position B, which is closer to the tower, she finds that the angle has become 59°. Find the distance that the woman walked between making the observations.

Big triangle (A)

$$\tan 37 = \frac{200}{A}$$

$$A = \frac{200}{\tan 37}$$

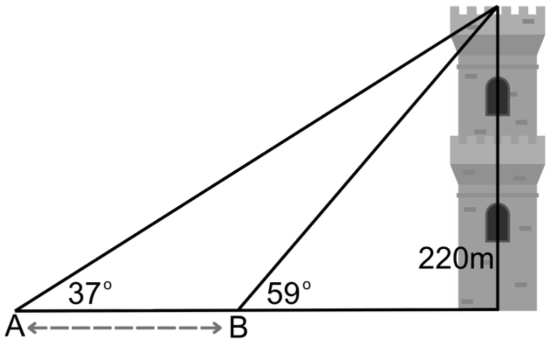
$$A = 265.41m$$

Small triangle (B)

$$\tan 59 = \frac{200}{B}$$

$$B = \frac{200}{\tan 59}$$

$$B = 120.17m$$

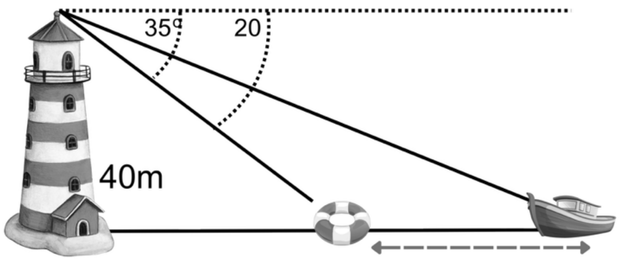


Distance between = 265.41 – 120.17 = 145.24

The woman walked 145.24 metres between observations.

With your teacher.

From the top of a lighthouse, 40m high, a lookout sees a small boat and a buoy in the same straight line out to sea. The angle of depression to the buoy is 35°. The angle of depression to the boat is 20°. Find how far apart the boat and the buoy are.



MEASUREMENT

2026 version

Jess Bertram

BERTRAM
EDUCATION