BERTRAM
EDUCATION

What is a

Teacher Edition?

A Teacher Edition contains everything in the student workbook , except it has worked solutions, tips, tricks, and planning tools written in red. See what your students can see as well as all the answers.

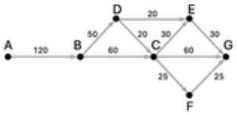
Note: the Teacher Edition replaces the student workbook, you do not need to purchase both.

Student workbook

With your teacher.

1. Find the maximum flow from source to sink in the following networks.

2. A highway network is represented by the graph below. Weights represent the capacity of different sections of the highway in cars per minute.
   a. Find the maximum flow of traffic from A to G.



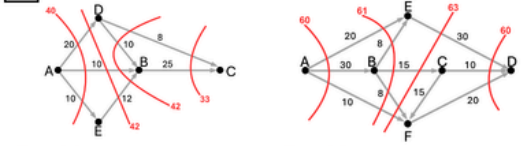
   b. Authorities would like to improve traffic flow but wish to improve only one section of road. Which sections will bring about improvement?

   c. By how much would the overall flow improve if the section DE was to be upgraded to 60 cars/min?

Teacher Edition

With your teacher.

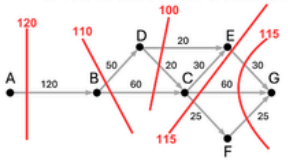
1. Find the maximum flow from source to sink in the following networks.



Network 1.
Minimum cut is 33 through DC and BC.
Maximum flow is 33 units.

Network 2.
Minimum cut is 60. Multiple minimum cuts indicate that the network contains more than one bottleneck. Even if one bottleneck is improved, another section may still limit the overall maximum flow.
Maximum flow is 60 units.

2. A highway network is represented by the graph below. Weights represent the capacity of different sections of the highway in cars per minute.
   a. Find the maximum flow of traffic from A to G.



Maximum flow of traffic from A to G = 100 cars per minute.

   b. Authorities would like to improve traffic flow but wish to improve only one section of road. Which sections will bring about improvement?

To improve overall traffic, authorities must improve roads that lie on the minimum cut. Any roads improved on the minimum cut (DE, DC or BC) will improve overall traffic. Improving roads NOT on the minimum cut will not improve traffic overall, as the bottleneck will remain.

   c. By how much would the overall flow improve if the section DE was to be upgraded to 60 cars/min?

Section DE being upgraded changes which cut is now the minimum. The new minimum cut is 110. **The overall flow has improved by 10 cars per minute.**

From analysing the overall percentages (on the previous page) we can see that:

- Overall, 52% of the TCE students play sport
- More year 12 students (62%) play sport than year 11 students (38%)
- Year 12 students are over-represented in the sample (62% of respondents)

We could also use percentages to divide the data by year level.

	Year 11	Year 12
Play sport	58%	48%
Don't play sport	42%	52%
Total	**100%**	**100%**

These percentages were calculated from the column totals. The number of year 11 students who play sports (11) was divided by the total number of year 11 students (19). $(11 \div 19 = 0.58 \times 100 = 58\%)$

From analysing the year level specific percentages, we can see:

- Among Year 11 students, most play sport (58%) but many do not.
- Among Year 12 students the group is more evenly split
- Suggests that Year 11's are *more likely* to play sport than Year 12 students.

With your teacher:

A community group wants to know how people prefer to spend their weekends. They ask community members whether they prefer outdoor activities (like hiking or mountain biking) or indoor activities (like movies or gaming). The responses are grouped by age: 'Under 20' and '20 & Over'

Complete the two-way frequency tables below by adding the totals to the first table, then converting to percentages in the second table.

	Under 20	20 & over	Total
Outdoor	15	22	**37**
Indoor	18	45	**63**
Total	**33**	**67**	**100**

	Under 20	20 & over	Total
Outdoor	**15%**	**22%**	**37%**
Indoor	**18%**	**45%**	**63%**
Total	**33%**	**67%**	**100%**

__RED__ text is a 'teacher only' answer.
The students cannot see this in their books.

With your teacher.

A survey was conducted with people about their preferred mode of transport to school or work. They could choose between public transport or private car. The responses are also grouped by gender: Male, female, and other.

a. Complete the table below

	Male	Female	Other	TOTAL
Public	5	10	2	**17**
Private	10	25	3	**38**
TOTAL	**15**	**35**	**5**	**55**

b. Turn the table from (a) into percentages out of the total (55)

	Male	Female	Other	TOTAL
Public	**9%**	**18%**	**4%**	**31%**
Private	**18%**	**45%**	**5%**	**69%**
TOTAL	**27%**	**64%**	**9%**	**100%**

c. Turn the table from (a) into percentages per gender.

	Male	Female	Other
Public	**33%**	**29%**	**40%**
Private	**67%**	**71%**	**60%**
TOTAL	**100%**	**100%**	**100%**

d. Write two observations about your findings from this data.

Suggested observations could be about:

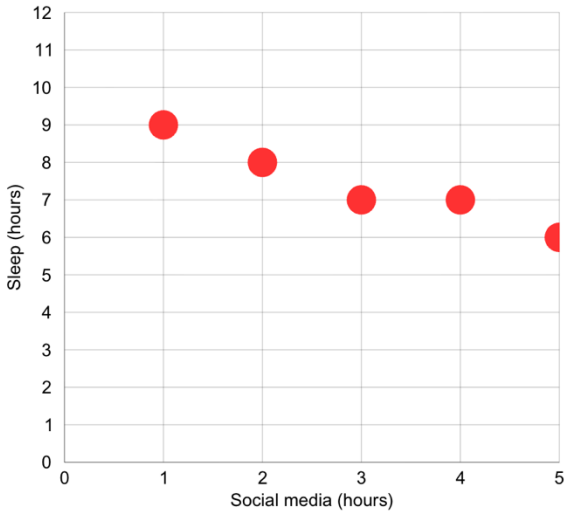
- Majority preferred private transport
- Preferences for public vs private transport were pretty similar across genders
- Specific – such as nearly a third of people surveyed identified as male

With your teacher:

A group of students recorded the number of hours they spent on social media each day and the number of hours they slept that night. They are curious if the time on social media impacts how much sleep they get.

Their results are as follows:

Social media (hours)	Sleep (hours)
1	9
2	8
3	7
4	7
5	6



Your tasks:

a. Determine which column is the independent variable (X axis) and which is the dependent variable (Y axis).

Social media = independent variable (x). Sleep depends on social media usage.

b. Prepare a scatterplot from scratch using the graph paper above.

See above.

c. Describe the association

There is a strong negative linear association between the hours spent using social media during the day and hours of sleep at night.

d. Write a few sentences interpreting the results in the context of the data.

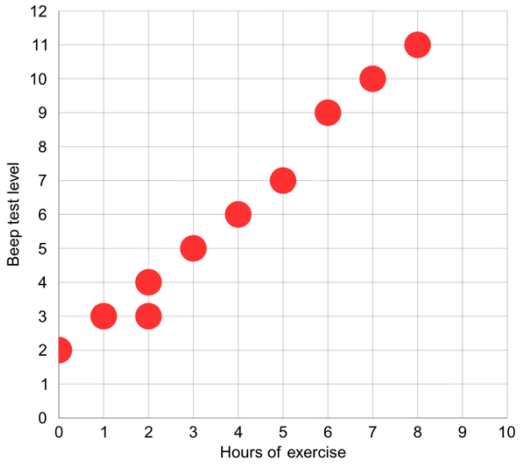
There is a strong negative linear association between the number of hours spent using social media during the day and hour of sleep that night. In context, students who used social media less during the day appeared to sleep better than those students who spent more time on social media.

With your teacher:

The fitness beep test results of 12 students are shown in the following table together with the number of hours they exercised in the week leading up to the test.

Hours of exercise	1	3	4	2	5	0	6	3	7	4	8	2
Beep test level	3	5	6	4	7	2	9	5	10	6	11	3

1. Plot the data on the scatterplot.
   See graph
2. What type of correlation do you think is involved?
   Strong positive correlation
3. Use your calculator to find the correlation coefficient.
   0.9911



4. Write a conclusion based on your correlation coefficient.
   There is a strong positive correlation between the hours of exercise leading up to the beep test and the level achieved in the test. The more hours exercised, the higher the beep test level.

With your teacher: Write a suitable conclusion based on the correlation coefficient for each of the following:

1. A psychologist studies the relationship between hours of sleep per night and student concentration scores on a memory test. She finds a correlation coefficient of **0.62**.
   There is a moderate positive correlation between the variables. There is some evidence to show that the greater number of hours sleep per night, the higher the scores achieved on the test.
2. A sports scientist investigates the relationship between the number of hours spent at soccer training per week and players' fitness test results. He finds a correlation coefficient of **0.81**.
   There is a strong positive correlation between the variables. The more hours spent at soccer training, the higher the fitness test results.

With your teacher:

1. Find the equation of the line that passes through the points (1,4) and (5,8)

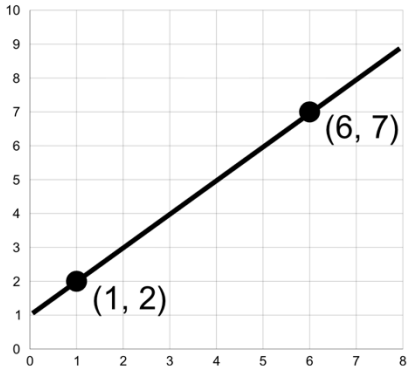
$$m = \frac{8-4}{5-1} = \frac{4}{4} = 1$$ ← *Find the gradient first*

$$y - 8 = 1(x - 5)$$

$$y - 8 = 1x - 5$$

$$y = x + 3$$ ← *Final equation*

2. Find the equation of the line below.



$$m = \frac{7-2}{6-1} = \frac{5}{5} = 1$$

$$y - 7 = 1(x - 6)$$

$$y - 7 = 1x - 6$$

$$y = x + 1$$

Finding the gradient and Y intercept from two points using Casio Classpad

1. On the main menu, tap the Statistics icon.
2. Enter your data (minimum of 2 points)
   - Type the x-values into List 1
   - Type the y-values into List 2
3. Tap Calc (top toolbar).
   - Choose regression – linear regression
   - Select List 1 as the X-list and List 2 as the Y-list.
   - Press ok
4. The calculator will display a variety of information.

   a = gradient

   b = y intercept

   r = correlation coefficient

   r^2 = coefficient of determination

CONSTRUCTING GRAPHS – SUBSTITUTION METHOD

There are multiple ways to draw a graph from a given equation. In this workbook we will focus on the substitution method. You may use a different method if you prefer.

Steps to graph a line – substitution method

1. Choose at least two values for (x)
2. Substitute the values for (x) into the equation to find (y)
3. Plot each pair (x,y)
4. Draw a straight line through the points

Worked example. Graph the equation $y = 2x + 1$

Step 1 – Choose 2 values for (x)

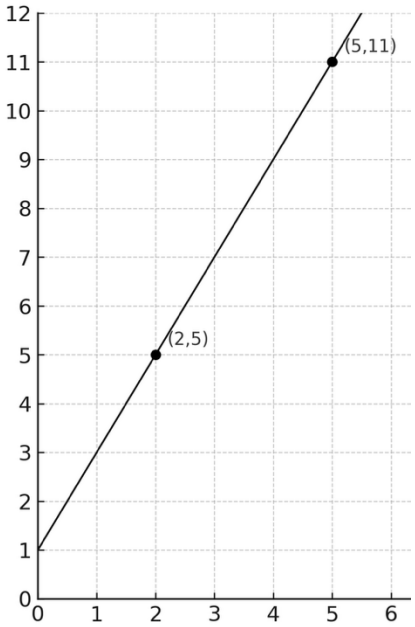
X = 2 and X = 5

Step 2 – Substitute each X into the equation.

(X = 2)	(X = 5)
Y = 2(2) + 1	Y = 2(5) + 1
Y = 5	Y = 11
Point @ (2,5)	Point @ (5,11)

Step 3 – Plot each pair

Step 4 – Draw a straight line through the points



With your teacher. Graph the equation $y = -3x + 10$

Step 1 – Choose 2 values for (x)

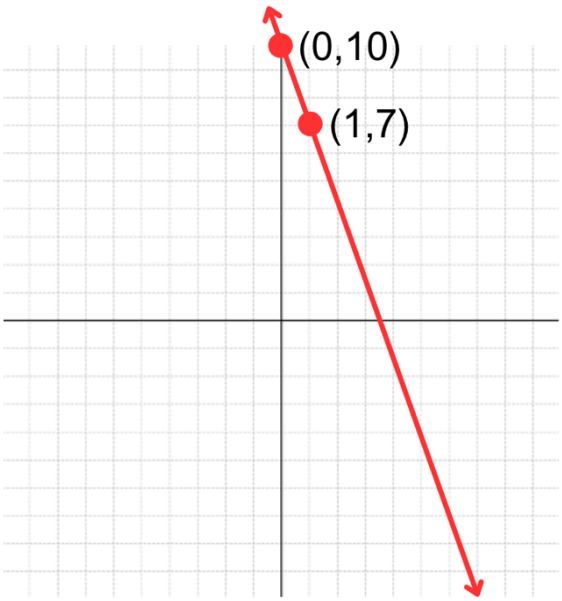
X = 0 and X = 1

Step 2 – Substitute each X into the equation.

(X = 0)	(X = 1)
Y = –3(0) + 10	Y = –3(1) + 10
Y = 10	Y = 7
Point @ (0,10)	Point @ (1,7)

Step 3 – Plot each pair

Step 4 – Draw a straight line through the points



As well as graphing the equation of a line, we can also use substitution to find the value of y for a specific value of x. This is very useful when we want to make predictions in real-life contexts.

Worked Example

Katy runs a small business. She has found that her profit (y, in dollars) can be modelled by the equation y = 15x – 50, where x is the number of items she sells.

She wants to know what her profit will be if she sells 20 items.

Substitute x = 20:

y = 15(20) - 50

y = 300 - 50

y = 250

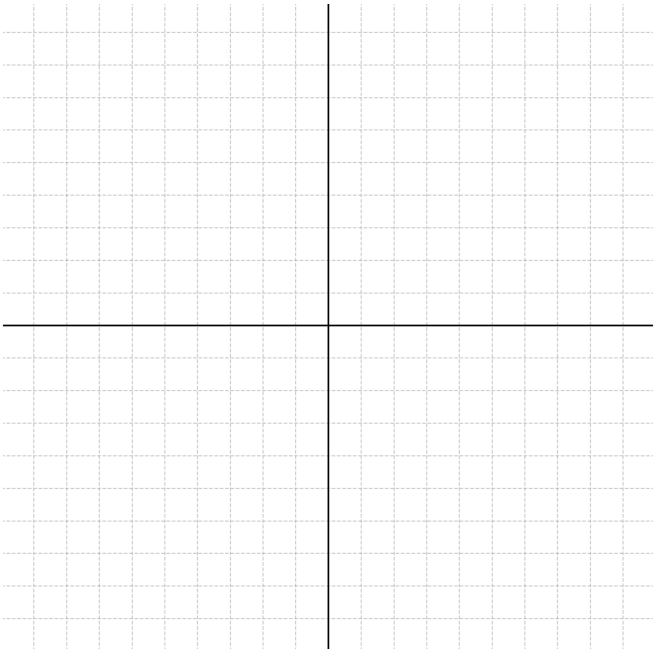
If Anna sells 20 items, she can expect to make a profit of $250.

With your teacher. Using the example above, find Katy's profit when she sells 10 items, 50 items, and 100 items.

Substitute x = 10:	**Substitute x = 50:**	**Substitute x = 100:**
y = 15(10) - 50	**y = 15(50) - 50**	**y = 15(100) - 50**
y = 150 - 50	**y = 750 - 50**	**y = 1,500 - 50**
y = 100	**y = 700**	**y = 1,450**

Try it yourself (SUBSTITUTION): *Ans pg77*

1. Graph the following equations on the same graph paper.
   a. $y = 3x - 4$
   b. $y = 5x + 1$
   c. $y = -2x + 4$
   d. $y = 0.5x + 0.5$
   e. $y = -x + 2$
2. For each of the above equations, find the value of Y when:
   X = 5
   X = 10
   X = 100



With your teacher. For the following data you are going to find the equation of the line of best fit. First by hand, then using your calculator.

x (Hours of Sleep)	4	5	5	6	6	7	7	8	8	9
y (Productivity Score)	45	50	54	56	61	65	70	72	76	79

By hand.

Points @ (5,52) and (9,80)

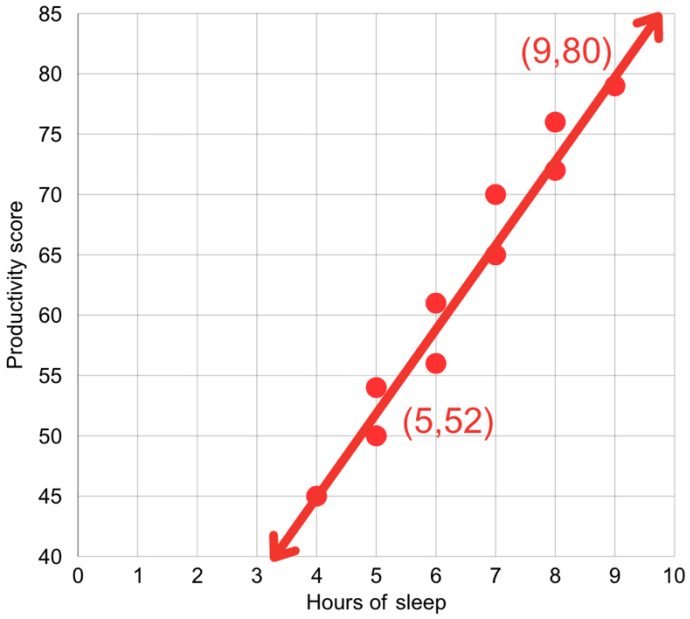
$$m = \frac{80-52}{9-5} = \frac{28}{4} = 7$$

$$y - 52 = 7(x-5)$$

$$y - 52 = 7x - 35$$

$$y = 7x + 17$$

There are many different answers for this kind of question. It will never be as accurate as finding the line of best fit with your calculator.



Using calculator.

$y = 7.11x + 16.58$

$a = 7.11, \quad b = 16.58, \quad r = 0.9796 \quad r^2 = 0.9597$

Interpret, in context, the values of a, b, r and r^2 found by your calculator.

The gradient ($a = 7.11$) tells us for every extra hour of sleep the participants productivity score went up by 7.11. The Y intercept ($b = 16.58$) tells us on no sleep the participant should score 16.58. Correlation coefficient ($r = 0.9796$) tells us there is a strong positive linear association between hours of sleep and productivity. The coefficient of determination ($r^2 = 0.9597$) tells us 95.97% of the variation in productivity can be explained by hours of sleep.

With your teacher.

Temperature (°C)	15	17	19	21	23	25	27	29	30
Ice creams sold	72	74	83	89	98	107	116	122	128

Ally owns an ice cream parlour and wants to predict sales throughout summer. She collects the above data over the course of 9 days and finds the line of best fit is $y = 3.89x + 9.74$. Help Ally interpret the results by answering the below questions.

1. What is the Y intercept and what does this represent?

The Y intercept of 9.74 means when the temperature outside is 0°C she should sell 9.74 ice-creams.

2. What is the gradient and what does it represent?

The gradient of 3.89 means that for every 1°C increase in outside temperature, Ally can expect to sell 3.89 more ice-creams.

3. What does it mean if (r) is 0.996 and (r^2) is 0.991?

The correlation coefficient (r = 0.996) tells us that there is a strong positive linear association between the temperature outside and the number of ice-creams sold. As temperature increases, ice-cream sales also increase.

The coefficient of determination (r^2 = 0.991) means that 99.1% of the variation in ice-cream sales can be explained by temperature. Only 0.9% of the variation is due to other factors (random, marketing, days of the week etc).

4. Ally wants to predict the number of ice-creams sold if the temperature was 24 degrees.
    a. Is this considered interpolation or extrapolation?
    b. How many ice-creams would be sold?
    c. Is this a reliable estimate?

It is interpolation, 24 °C is within the range of original data (15 to 30°C). Estimated 103.1 ice-creams sold when temperature is 24°C. With the strong correlation coefficient and coefficient of determination plus interpolation, this is considered a very reliable estimate.

5. Ally wants to predict the number of ice-creams sold if the temperature was 100 degrees.
    a. Is this considered interpolation or extrapolation?
    b. How many ice-creams would be sold?
    c. Is this a reliable estimate?

This is extrapolation. There would be a predicted nearly 399 ice-creams sold if the temperature was 100°C. It is NOT a reliable estimate. It is extrapolation (outside the given data range) so is unreliable. If it was 100 degrees people would not be outside buying ice-cream.