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# 2027 Sample pack

# General Maths level 3

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# 2027 sample pack

Discover how the course is structured through explicit theory, worked examples, guided practice and independent practice questions.
This sample pack includes select pages from each of the 5 units in the General Maths level 3 course.

**Please note:** The 2027 editions have been substantially revised and updated. To ensure page numbers, questions and teacher resources align, it is recommended that all students within a class use the same edition.

# 2027 available courses.

- General mathematics (level 2)
- General mathematics (level 3)

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# Student workbooks

## Overview

- Comprehensive workbook set covering the entire school year.
- Structured around the gradual release model (I do, We do, You do)

**WHAT IS AN ARITHMETIC SEQUENCE?**

An arithmetic sequence is a pattern where the numbers change by the same amount each step. This amount can be:

| | | | |
|---|---|---|---|
| Positive | (increasing sequence) | → 4, 7, 10, 13, 16, … | (+3 each time) |
| Negative | (decreasing sequence) | → 15, 10, 5, 0, -5 … | (–5 each time) |
| Zero | (constant sequence) | → 8, 8, 8, 8, … | (no change, +0) |

**Arithmetic sequences add or subtract the SAME number each time.**

Arithmetic sequences model situations where something increases or decreases by a fixed amount over time. These sequences can represent real life situations.

| | |
|---|---|
| Saving $50 every week | → 50, 100, 150, 200, … |
| Temperature drops 2°C each hour | → 18, 16, 14, 12, … |
| A plant grows 3 cm each day | → 5, 8, 11, 14, … |

**Theory** – Concepts are introduced using clear, student-friendly language.

**Worked example. From sequence → graph**

Jasmin starts a new job and earns $200 in her first week. Each week, she earns $50 more than the previous week as she takes on extra responsibilities.

This sequence could be written as: 200, 250, 300, 350, 400, 450, …

**Step 1:** Make a table of term position (n) and term value ($t_n$) for the first 6 terms.

| Term position (n) | Value ($t_n$) | | Point @ (n, $t_n$) |
|---|---|---|---|
| 1 | 200 | → | (1, 200) |
| 2 | 250 | → | (2, 250) |
| 3 | 300 | → | (3, 300) |
| 4 | 350 | → | (4, 350) |
| 5 | 400 | → | (5, 400) |
| 6 | 450 | → | (6, 450) |

**Step 2:** Plot the points. We only plot the points (discrete values). We do not join them with a continuous line because sequences only exist at set intervals.

**I do** – Worked examples introduce each concept and model required skills.

**With your teacher.**

1. Identify if each sequence is arithmetic. If it is, state the common difference (d).

| | | Arithmetic sequence? | Common difference (d) |
|---|---|---|---|
| a) | 3, 6, 9, 12, … | | |
| b) | 10, 7, 4, 1, … | | |
| c) | 5, 10, 20, 40, … | | |
| d) | 5, 5, 5, 5, … | | |
| e) | 2, 5, 9, 14, … | | |
| f) | 100, 90, 80, 70, … | | |

2. Write the first 5 terms ($t_1$, $t_2$, $t_3$, $t_4$, $t_5$) of each arithmetic sequence.
   a) Start at 4, add 6 each time

**We do** – Guided practice questions for students to complete with their teacher.

**Try it yourself (INTRO TO SEQUENCES)**

1. State whether each of the following sequences are arithmetic. If they are, state the common difference (d), first term ($t_1$) and the third term ($t_3$)
   a) 9, 12, 15, 18, …
   b) 16, 8, 4, 2, …
   c) 5, 3, 1, –1, –3 …
   d) 2, 5, 9, 14, …
   e) –10, –5, 0, 5, …
   f) 7, 7, 7, 7, …
2. Fill in the missing terms.
   a) 2, __, __, 8, 10
   b) __, 14, 11, 8, __
   c) –3, __, 1, __, 5
   d) 50, 45, __, 35, __

**You do** – Independent practice questions to build confidence and skill

# GENERAL MATHEMATICS

## Level 3

2027 EDITION

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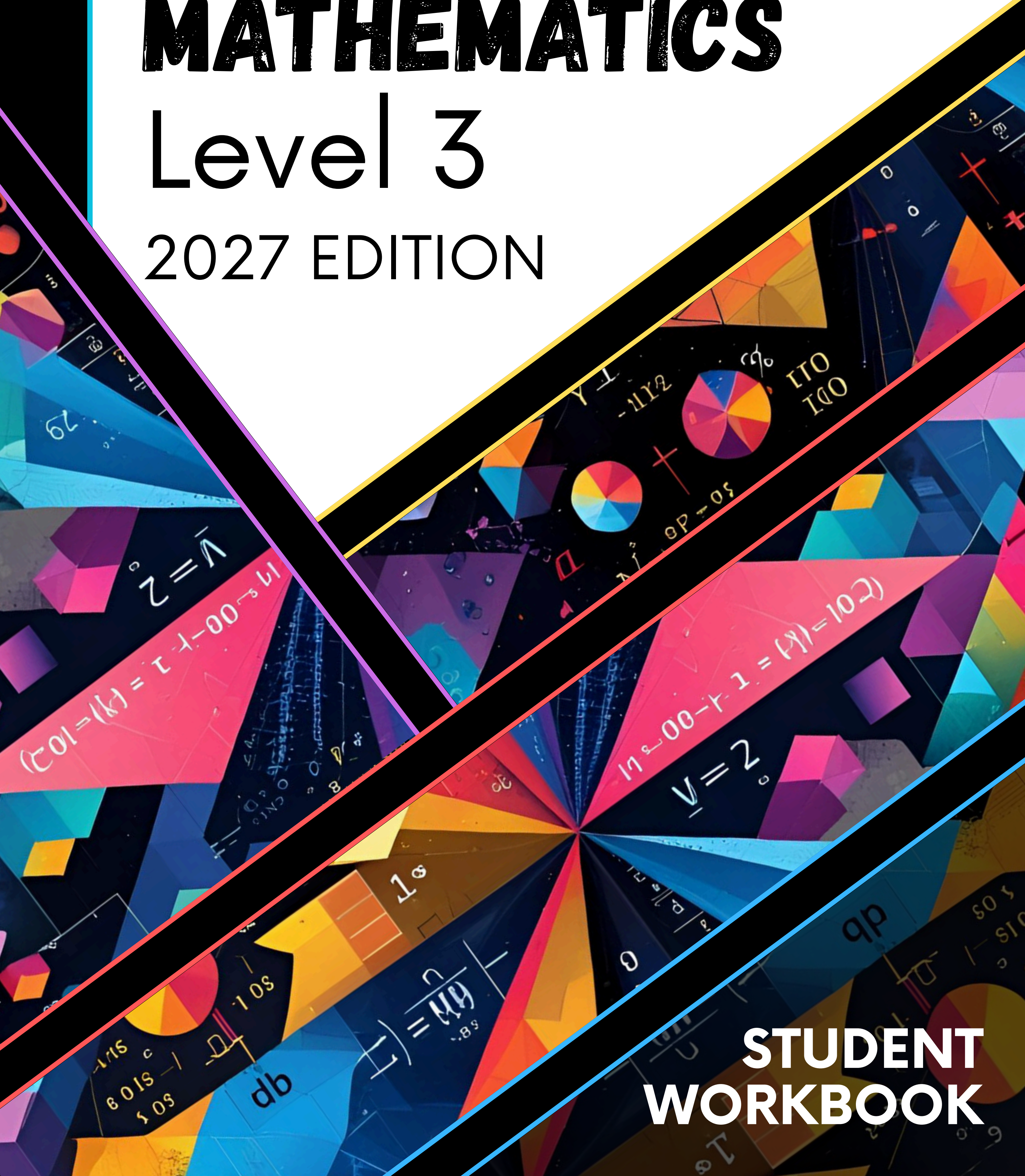


STUDENT WORKBOOK

2027

# GENERAL MATHEMATICS

## Level 3

# BIVARIATE DATA

## STRAIGHT LINE EQUATION INTRODUCTION

A straight line has the general equation:

$$\boldsymbol{y = ax + b}$$

*(can also be written as* $\boldsymbol{y = mx + c}$*)*

where:

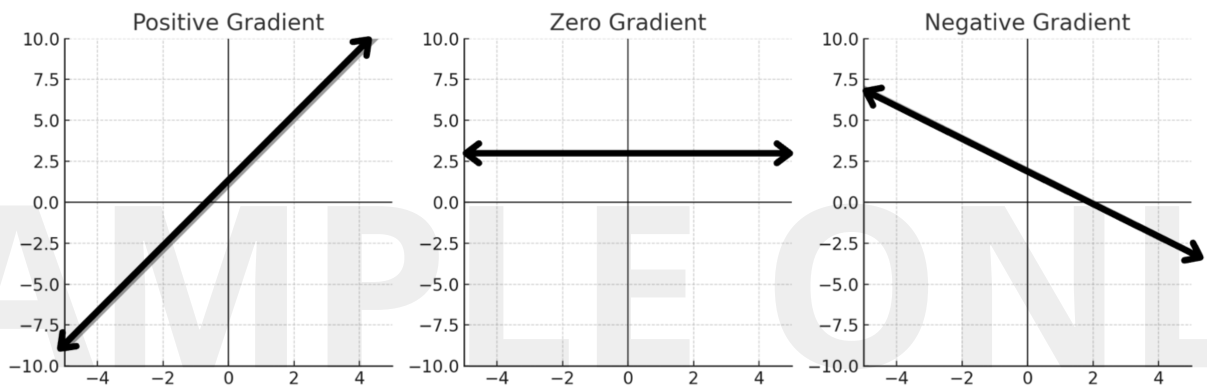
- **a** is the **gradient (slope)** of the line
- **b** is the **y-intercept** (the point where the line crosses the y-axis)

### Gradient (slope)

The gradient tells us how steep the line is.

The gradient can be positive, zero, or negative.

The larger the number, the steeper the graph.






We can find the **gradient of a line** from two points.

$$\frac{rise}{run} \quad OR \quad \frac{y_2 - y_1}{x_2 - x_1} \quad OR \quad \frac{\Delta\, y}{\Delta\, x}$$

There are lots of different ways to write it, but essentially gradient is calculated by the difference in Y values divided by the difference in X values.

**Worked examples.**

Find the gradient of the line between each pair of points.

a) (2,3) and (6,11)

$$m = \frac{11-3}{6-2}$$

$$m = \frac{8}{4}$$

$$m = 2$$

b) (−4,5) and (2, −1)

$$m = \frac{-1-5}{2--4}$$

$$m = \frac{-6}{6}$$

$$m = -1$$

c) (0,−2) and (3,−2)

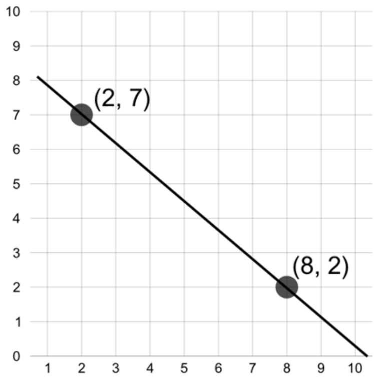
$$m = \frac{-2--2}{3-0}$$

$$m = \frac{0}{3}$$

$$m = 0$$

**With your teacher:** Find the gradient between the following pairs of points.

(1, 2) & (5, 6) (1, 5) & (4, -1) (1, 2) & (5, 3)




### The Y intercept

Once we know the gradient of a line, we can find the y-intercept by substituting one of the points into the straight-line equation (y = mx + c). The steps are:

1. Find the gradient ($m$) using $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Use the gradient and one of your points in the formula $y - y_1 = m(x - x_1)$



**Worked Examples.**

1. Find the equation of the line through the points (2, 3) and (6, 11).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 3}{6 - 2} = \frac{8}{4} = 2$$ *← Find the gradient first*

$$y - 11 = 2(x - 6)$$

$$y - 11 = 2x - 12$$

$$y = 2x - 1$$ ***← Final equation***

2. Find the equation of the line through (-4, 5) and (2, -1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{2 - -4} = \frac{-6}{6} = -1$$ *← Find the gradient first*

$$y - 5 = -1(x - -4)$$

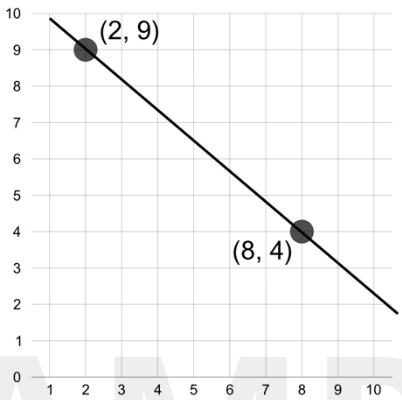
$$y - 5 = -1x - 4$$

$$y = -x + 1$$ ***← Final equation***

**With your teacher:**

1. Find the equation of the line that passes through the points (1,4) and (5,8)

2. Find the equation of the line below.






**Finding the gradient and Y intercept from two points using Casio Classpad**

1. On the main menu, tap the Statistics icon.
2. Enter your data (minimum of 2 points)
    - Type the x-values into List 1
    - Type the y-values into List 2
3. Tap Calc (top toolbar).
    - Choose regression – linear regression
    - Select List 1 as the X-list and List 2 as the Y-list.
    - Press ok
4. The calculator will display a variety of information.

    **a = gradient**

    **b = y intercept**

    r = correlation coefficient

    $r^2$ = coefficient of determination

**Try it yourself (STRAIGHT LINE INTRODUCTION).**

1. State the Y intercept (c) and gradient (m) in each equation.

a) $y = 3x + 4$ b) $y = 2x + 5$ c) $y = 6x + 3$ d) $y = 9x + 2$

e) $y = 3x - 5$ f) $y = 7x - 10$ g) $y = 10x - 6$ h) $y = 5x - 4$

i) $y = -2x + 5$ j) $y = -3x + 10$ k) $y = -4x - 10$ l) $y = -5x - 5$

m) $y = 5 + 2x$ n) $3x - 0.5 = y$ o) $y = \frac{x}{2} + \frac{3}{2}$ p) $k = 2r - 5.3$

2. Write the equation of a line given the gradient (m) and Y intercept (c).

a) m = 1, c = 0 b) m = 2, c = 3 c) m = -1, c = 4 d) m = 5, c = -2

e) m = 0, c = 6 f) m = -2, c = -3 g) m = 3, c = 1 h) m = 0.5, c = -1

i) m = -4, c = 2 j) m = 7, c = -5 k) m = -0.5, c = 3 l) m = 10, c = 0

m) m = 2/3, c = -4 n) m = -3/2, c = 5 o) m = 8, c = -10 p) m = -6, c = -7

3. Find the gradient of the straight line passing through the following points.

a) (0,0) and (4,4) b) (1,2) and (3,6) c) (2,5) and (4,2) d) (-1,3) and (2,3)

e) (0,5) and (5,0) f) (-2,-2) and (2,2) g) (1,1) and (4,7) h) (2,8) and (6,2)

i) (3,5) and (3,10) j) (0,-4) and (6,2) k) (-3,7) and (5,-1) l) (2,3) and (5,3)

m) (1,2) (3.5,6.5) n) (0,0) and (7,10) o) (-4,-2) and (4,6) p) (2,7) and (6,3)



4. Find the equation of the line passing through the points.

a) (0,0) and (2,2) b) (1,1) and (3,5) c) (0,2) and (4,6) d) (2,3) and (6,3)

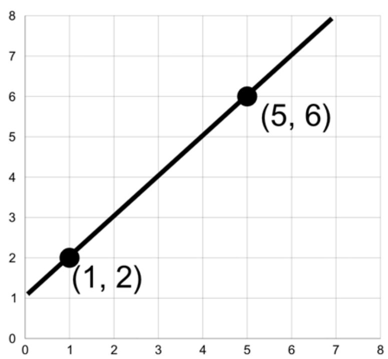
e) (-2,1) and (2,5) f) (0,-1) and (4,3) g) (-3,-2) and (3,4) h) (1,4) and (5,-2)

i) (-2,6) and (2,0) j) (0,5) and (5,0) k) (1.5,2), (3.5,6) l) (-1,3.4), (2,7.4)

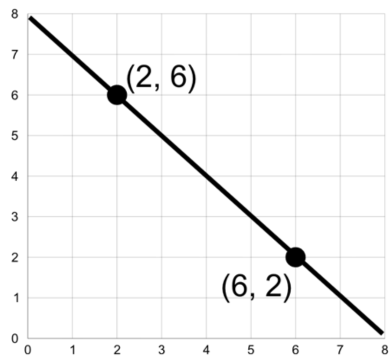
m) $(0,\frac{3}{2})$ and $(4,\frac{11}{2})$ n) $(\frac{2}{3},1)$ and $(\frac{8}{3},5)$ o) (-4.5,-2), (3.5,5) p) (-2,7), (6,-1)

5. Find the equation of the line in the following graphs:

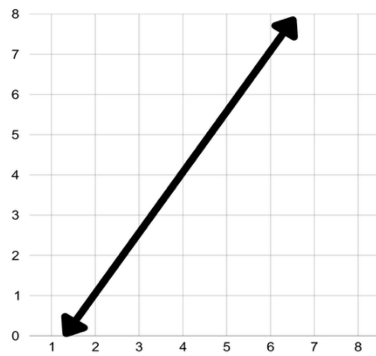
a)




b)

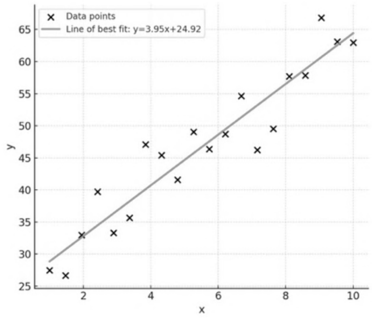



c)



## LINE OF BEST FIT

When we have a scatterplot showing an association between two variables, we can summarise the relationship using a **line of best fit.** This line is not drawn through all the points - it is placed so that it best represents the overall trend of the data.




Finding the line of best fit can be done in 3 ways;

1. Fit a line by eye and find the equation using the two-point method.
2. The regression method
3. Using a CAS calculator

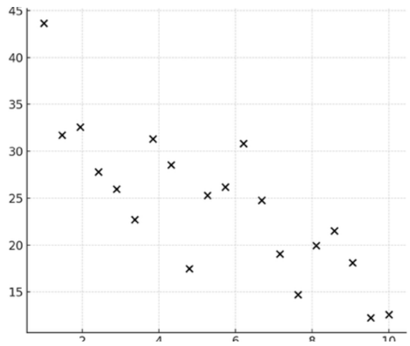
**Method 1: Fit the line by eye and find equation using two-point method**

1) Plot the scatterplot.
2) Use a ruler to draw a straight line that seems to follow the trend of the data.
3) The line should be fitted in a way that minimises the total vertical distance between the line and all the raw data points.
4) Choose two points on the line you've just drawn (not the original points), and use them to calculate the equation in the form y = ax + b

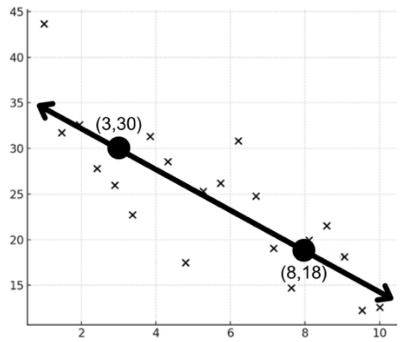


**Worked example.**

**Step 1:** Plot the scatterplot



**Step 2:** Fit the line




**Step 3:** Use two points (3,30) and (8,18) to calculate the equation of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18 - 30}{8 - 3} = \frac{-12}{5} = -2.4$$ ← *Find the gradient first*

$$y - 30 = -2.4(x - 3)$$

$$y - 30 = -2.4x + 7.2$$

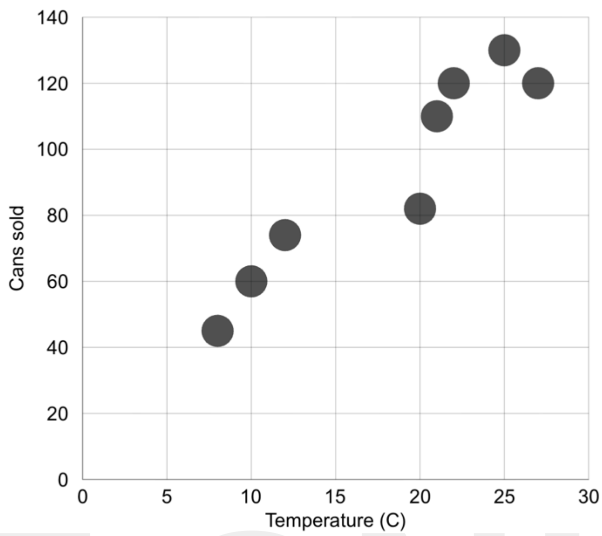
$$y = -2.4x + 37.2$$ ← ***Equation of the line of best fit***

*Note: This is the least accurate of the 3 methods.*

**With your teacher.**

1. The owner of a soft drinks vending machine records the daily temperature and the number of cans of soft drink sold each day over an 8-day period.
The data is then graphed, with 'Temperature' as the independent variable (x axis) and 'cans sold' as the dependent variable (y axis) as the number of cans sold is responding to the daily temperature (hotter weather = more drinks sold)

| Daily Temp °C | No. of cans sold |
|---|---|
| 20 | 82 |
| 22 | 120 |
| 10 | 60 |
| 8 | 45 |
| 25 | 130 |
| 27 | 120 |
| 21 | 110 |
| 12 | 74 |






a) Fit the trend line by eye and use the two-point method to find its equation.

b) Check the line of best fit appears reasonable when compared with the graph. Is the graph sloping in the right direction? Is the position of the y intercept confirmed by the graph?

**With your teacher.**

2. The management of a forest walk tourist attraction adjusts the visitor entry price in order to see how it effects the number of visitors

| Entry Price | Visitors |
|---|---|
| \$ 3.50 | 80 |
| \$ 4.00 | 72 |
| \$ 4.50 | 60 |
| \$ 5.00 | 51 |
| \$ 5.50 | 53 |
| \$ 6.00 | 39 |
| \$ 6.50 | 31 |
| \$ 7.00 | 24 |

a) Prepare a scatter plot of the data.



b) Fit the trend line by eye and use the two-point method to find its equation.

c) What is the significance of the gradient and y intercept of your equation?

**Method 2: The regression method**

The regression method is a more accurate method for finding the equation of the line of best fit by use of a formula. The formula places the fitted line in a position that minimises the total distance of all points from the line.

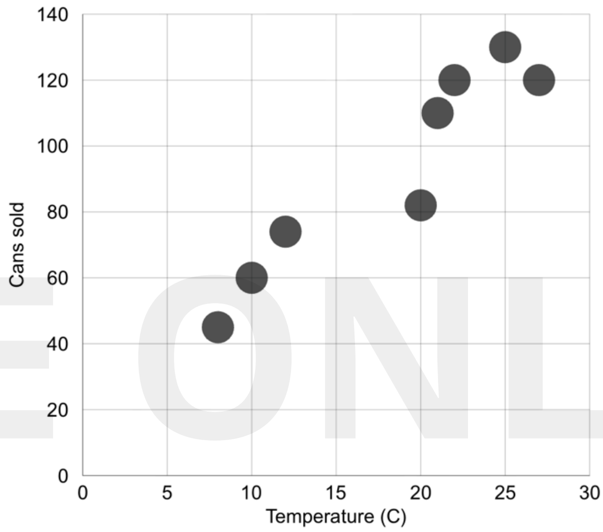
$y = ax + b$ where; $a = \dfrac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$ and $b = \dfrac{\sum y - a\sum x}{n}$

The formula looks formidable but is easier to manage if a tabular approach is used.

**Worked example.**

We will use the sale of soft drink cans example shown in method 1.

| Daily Temp °C | No. of cans sold |
|---|---|
| 20 | 82 |
| 22 | 120 |
| 10 | 60 |
| 8 | 45 |
| 25 | 130 |
| 27 | 120 |
| 21 | 110 |
| 12 | 74 |




**Step 1.** Add 2 additional columns to the data table.

The first column $(xy)$ multiplies the x and y data pairs.

The second column $(x^2)$ squares the x data (daily temperature).

| | Daily Temp °C $(x)$ | No. of cans sold $(y)$ | $xy$ | $x^2$ |
|---|---|---|---|---|
| | 20 | 82 | 1,640 | 400 |
| | 22 | 120 | 2,640 | 484 |
| | 10 | 60 | 600 | 100 |
| | 8 | 45 | 360 | 64 |
| | 25 | 130 | 3,250 | 625 |
| | 27 | 120 | 3,240 | 729 |
| | 21 | 110 | 2,310 | 441 |
| | 12 | 74 | 888 | 144 |
| **Totals $(\sum)$** | **145** | **741** | **14,928** | **2,987** |

**Worked example (continued).**

The final row in the table shows the sum total of each column.

The Greek letter sigma ($\sum$) means "add together all the…"

So the total of the column $x$ can be written as $\sum x$ (all the $x$ values added together).

The total of column $y$ could be written as $\sum y$ and so on.

| | | | | |
|---|---|---|---|---|
| **Totals ($\sum$)** | **145** | **741** | **14,928** | **2,987** |
| | $\sum x$ | $\sum y$ | $\sum xy$ | $\sum x^2$ |

**Step 2.** Substitute your values from the table into the formula to find the gradient ($a$).

In this example, there were 8 pieces of data, so $n = 8$.

Gradient $a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{8(14{,}928) - (145)(741)}{8(2{,}987) - (145)^2} = \frac{11{,}979}{2{,}871} = 4.172$

**Step 3.** Substitute your values into the formula to find the y intercept ($b$)

y intercept. $b = \frac{\sum y - a\sum x}{n} = \frac{741 - (4.172)(145)}{8} = \frac{136.06}{8} = 17.01$



**Step 4.** Bring the gradient and y intercept together to find the line of best fit.

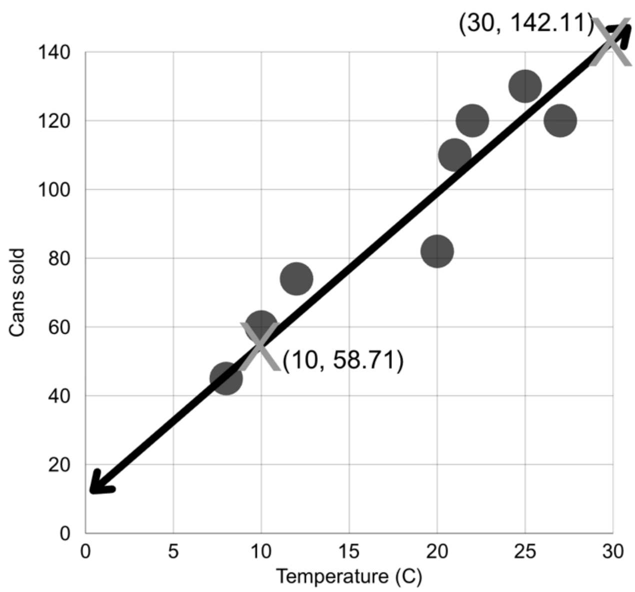
$y = 4.172x + 17.01$

**Step 5.** Using your equation, plug in two random values for $x$ to find points.

Use these points to add the line of best fit to your graph.

*Point 1.* When $x = 10$ $y = 4.172(10) + 17.01 = 58.71$ Point @ (10, 58.71)

*Point 2.* When $x = 30$ $y = 4.172(30) + 17.01 = 142.11$ Point @ (30, 142.11)



**With your teacher.**

A university bookshop finds that the number of economics textbooks sold each year is related to the number of students who enrol into the Economics class.

You are required to:

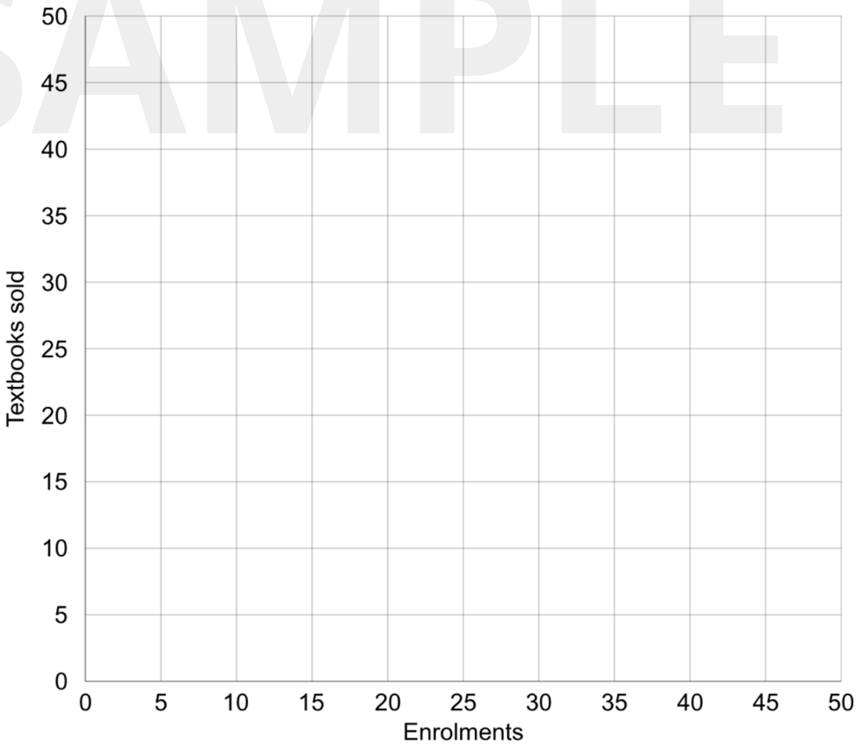
a) Plot the data on the below graph paper

b) Use the formulas on your formula sheet to determine the linear regression line

c) Position the linear regression line on the graph

| | Enrolments | Textbooks sold | *xy* | $x^2$ |
|---|---|---|---|---|
| | 20 | 25 | | |
| | 32 | 36 | | |
| | 41 | 40 | | |
| | 45 | 42 | | |
| | 26 | 27 | | |
| | 38 | 33 | | |
| Σ | | | | |





A calculator or spreadsheet can calculate the line of best fit more precisely.

This is called the least-squares line because it minimises the total squared vertical distances between the data points and the line.

**Method 3. Casio Classpad (least-squares)**

1. On the main menu, tap the Statistics icon.
2. Enter your data (all your raw data)
    - Type the x-values into List 1
    - Type the y-values into List 2
3. Tap Calc (top toolbar).
    - Choose regression – linear regression
    - Select List 1 as the X-list and List 2 as the Y-list.
    - Press ok
4. The calculator will display a variety of information.

    **a = gradient**

    **b = y intercept**

    r = correlation coefficient

    $r^2$ = coefficient of determination

    Equation of the line → $y = ax + b$

*$r^2$ measures how closely the plotted data points lie to the fitted trend line. It can be used to measure the reliability of predictions made.*



**Worked example.** Find the equation of the line of best fit for the following data.

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Y | 12 | 14 | 13 | 16 | 18 | 19 | 20 | 25 | 24 | 30 | 25 | 27 | 33 |

Following the steps in the box above – the screen should show the following:

$a$ = 1.6428571 ← *Gradient*

$b$ = 9.7307692 ← *Y intercept*

$r$ = 0.958826 ← *There is a strong, positive association between the data.*

$r^2$ = 0.9193472 ← *91.93% of the variation in Y-values can be explained by X.*

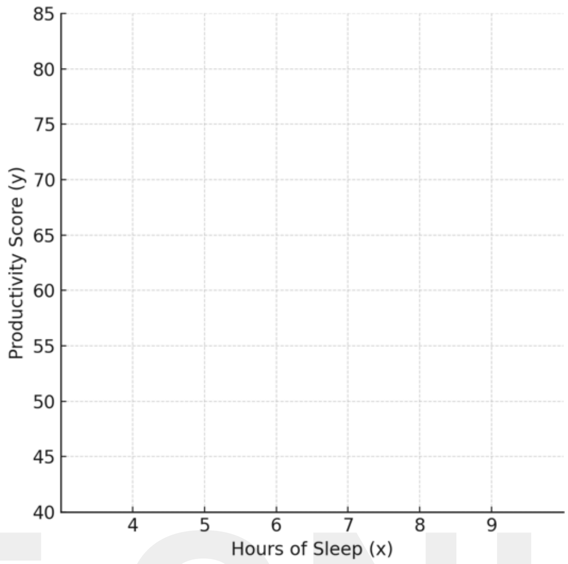
**Final equation:** $\boldsymbol{y = 1.64x + 9.73}$

Once you've written down the information, hit 'ok' and the calculator will show you the scatterplot with line of best fit drawn on it.

**With your teacher.** For the following data you are going to find the equation of the line of best fit. First by hand, then using your calculator.

| x (Hours of Sleep) | 4 | 5 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| y (Productivity Score) | 45 | 50 | 54 | 56 | 61 | 65 | 70 | 72 | 76 | 79 |

**By hand.**





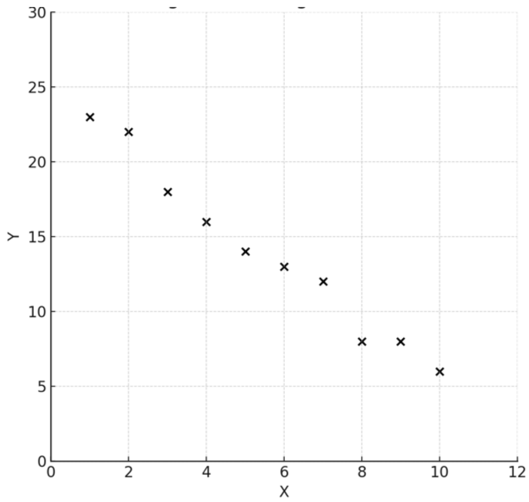

**Using calculator.**

**Interpret, in context, the values of a, b, r and $r^2$ found by your calculator.**

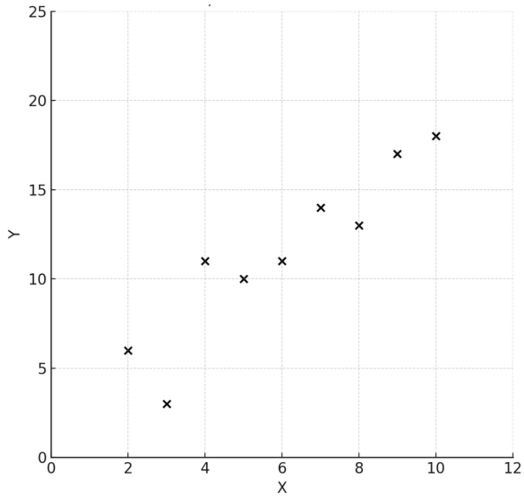
## Try it yourself (LINE OF BEST FIT).

1. For each scatterplot below, mark the line of best fit then find the equation.

a)

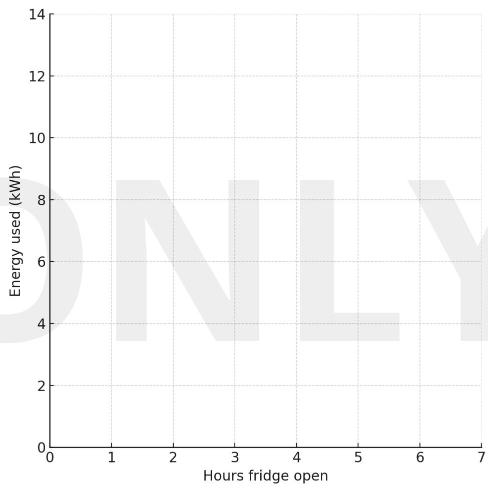



b)




2. The table shows the number of hours a fridge is left open and the energy used.

| Hours fridge open | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Energy used (kWh) | 2 | 3 | 5 | 7 | 8 | 10 | 12 |

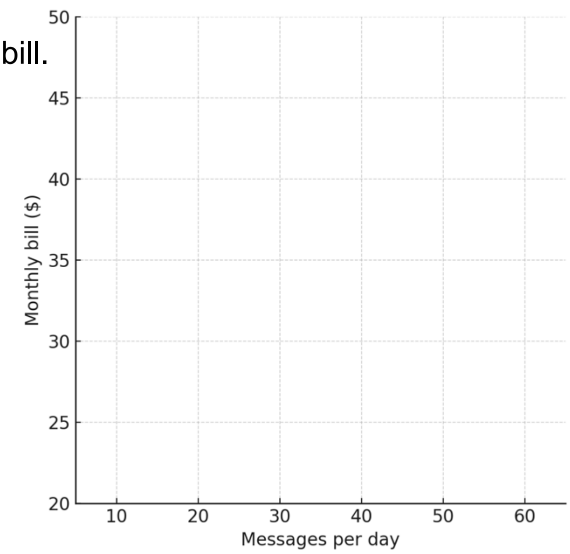



a) Plot a scatterplot of the data.
b) Draw a line of best fit by eye.
c) Use two points on the line to calculate the equation.
d) From your graph, roughly how much energy do you think would be used if the fridge was open for 2.5 hours?



3. A study records the number of text messages students send per day and their monthly phone bill.

| Messages per day | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| Monthly bill ($) | 25 | 28 | 32 | 38 | 41 | 46 |

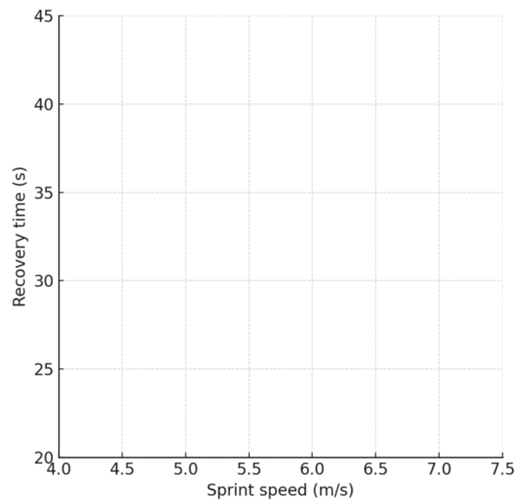



a) Plot the data.
b) Mark a line of best fit.
c) Find the equation of the line of best fit.

**Try it yourself (LINE OF BEST FIT cont).**

4. A coach records players' sprint speed and recovery time.

| Sprint speed (m/s) | 4.5 | 5.0 | 5.5 | 6.0 | 6.5 | 7.0 |
|---|---|---|---|---|---|---|
| Recovery time (s) | 42 | 38 | 35 | 31 | 29 | 26 |




   a) Draw a scatterplot.
   b) What type of correlation exists?
   c) Use two points to find the equation of the line of best fit.

5. The table shows the weight of apples purchased and their cost. Use your calculator to find the equation of the line, then copy a rough sketch of the graph.

| Weight (kg) | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| Cost ($) | 0 | 6 | 12 | 18 | 24 | 30 |



6. A car rental company records the distance travelled and the total rental cost. Use your calculator to find the equation of the line of best fit.

| Distance (km) | 0 | 100 | 200 | 300 | 400 | 500 |
|---|---|---|---|---|---|---|
| Cost ($) | 50 | 80 | 110 | 140 | 170 | 200 |

7. The number of cups of coffee consumed per day is compared with hours of sleep per night. The data is collected over two nights.

| Coffee (cups) | 0 | 1 |
|---|---|---|
| Sleep (hours) | 8 | 7.5 |

   a) Plot the scatterplot.
   b) Draw a line of best fit.
   c) Find the equation using two points.
   d) Do you this this line of best fit is an accurate representation of the association between cups of coffee and sleep? Why/why not?

8. The data shows the distance travelled by an electric scooter and battery percentage remaining. Use your calculator to find the line of best fit.

| Distance (km) | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| Battery (%) | 100 | 92 | 85 | 77 | 70 | 62 |

2027

# GENERAL MATHEMATICS

## Level 3

# SEQUENCES

Unit 2
SEQUENCES

# PART 1: INTRO TO SEQUENCES

## INTRODUCTION TO SEQUENCES

A sequence is a list of numbers in a specific order that follow a pattern.

**Examples of sequences**

2, 5, 8, 11, 14, … ← Add 3 each time

100, 50, 25, 12.5, … ← Divide by 2 each time (or multiply by 0.5)

50, 55, 60.5, 66.55, … ← Increase by 10% each time

**Why are sequences useful?**

Sequences are used to model change over time.

They help us answer questions like:

- How fast is something growing or shrinking?
- What will happen in the future?
- What will the total be after many steps?
- Will the values get very large, very small, or level off?



Each step in a sequence can represent a moment in time. Think of:

- Day 1, Day 2, Day 3…
- Year 1, Year 2, Year 3…
- Measurement 1, 2, 3…

At each step, something changes by a set amount.

That pattern is the rule of the sequence.

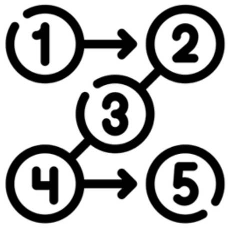
**If you can describe how the numbers change, you have a sequence.**

This workbook covers 3 different types of sequences.

1. Arithmetic sequences
2. Geometric sequences
3. Combination relations

We'll begin with simple number patterns and gradually build towards powerful mathematical models used to describe growth, decay, and change in the real world.

## ARITHMETIC VS GEOMETRIC SEQUENCES




Not all sequences change in the same way.

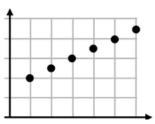
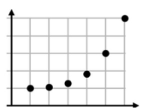
There are two main types of patterns we will learn:

1. Arithmetic sequences
2. Geometric sequences

| Arithmetic sequences | Geometric sequences |
|---|---|
| **Add or subtract** the same amount<br><br>Example:<br>2, 5, 8, 11, 14, … (+3)<br><br>The change (+3) is known as a common difference (d)<br><br>This creates linear change (straight-line pattern). | **Multiply** by the same amount<br><br>Example:<br>25, 50, 100, 200, … (×2)<br><br>The change (×2) is known as a common ratio (r)<br><br>This creates exponential change (curved pattern). |



**Side by side comparison:**

| ***Feature*** | **Arithmetic Sequence** | **Geometric Sequence** |
|---|---|---|
| *How it changes* | Add or subtract a constant | Multiply by a constant |
| *Example* | 4, 9, 14, 19, 24,… (+5) | 1, 4, 16, 64, 256, … (× 4) |
| *Change name* | Common difference (d) | Common ratio (r) |
| *Growth type* | Linear | Exponential |
| *Graph shape* | Straight line | Curve |
| *Example graph* |  |  |

**With your teacher.**

Identify the pattern in each sequence and mark as arithmetic (A) or geometric (G).

a) 4, 6, 8, 10, 12…  b) 5, 10, 15, 20, 25…  c) 10, 20, 40, 80, 160,…

d) 1, 3, 9, 27, 81,…  e) 9, 18, 27, 36, 45,…  f) 3, 6, 12, 24, 48,…

Unit 2
SEQUENCES

# PART 2: ARITHMETIC SEQUENCES

## WHAT IS AN ARITHMETIC SEQUENCE?

An arithmetic sequence is a pattern where the numbers change by the same amount each step. This amount can be:

Positive (increasing sequence) → 4, 7, 10, 13, 16, … (+3 each time)

Negative (decreasing sequence) → 15, 10, 5, 0, -5 … (–5 each time)

Zero (constant sequence) → 8, 8, 8, 8, … (no change, +0)

**Arithmetic sequences add or subtract the SAME number each time.**

Arithmetic sequences model situations where something increases or decreases by a fixed amount over time. These sequences can represent real life situations.

Saving \$50 every week → 50, 100, 150, 200, …

Temperature drops 2°C each hour → 18, 16, 14, 12, …

A plant grows 3 cm each day → 5, 8, 11, 14, …

You walk 1000 extra steps each day → 3000, 4000, 5000, 6000, …

**Important terminology**

| Terminology | Meaning | Example: 4, 7, 10, 13, … |
|---|---|---|
| **Term** | A value in the sequence | 7 is the 2nd term |
| **First term ($t_1$)** | The starting value | $t_1$ = 4 |
| **Common difference (d)** | The amount added/subtracted | d = +3 |
| **n** | The position of a term | 1st , 2nd , 3rd … |
| $t_n$ | The value of the nth term | $t_1$ = 4, $t_2$ = 7… |

You can represent terms with subscripts:

$t_1$ = first term, $t_2$ = second term, $t_3$ = third term, $t_{99}$ = 99th term, $t_n$ = nth term

**Remember: If a sequence changes by adding or subtracting the same amount each time, it is an arithmetic sequence.**

**With your teacher.**

1. Identify if each sequence is arithmetic. If it is, state the common difference (d).

| | | Arithmetic sequence? | Common difference (d) |
|---|---|---|---|
| a) | 3, 6, 9, 12, … | | |
| b) | 10, 7, 4, 1, … | | |
| c) | 5, 10, 20, 40, … | | |
| d) | 5, 5, 5, 5, … | | |
| e) | 2, 5, 9, 14, … | | |
| f) | 100, 90, 80, 70, … | | |

2. Write the first 5 terms ($t_1$, $t_2$, $t_3$, $t_4$, $t_5$) of each arithmetic sequence.

   a) Start at 4, add 6 each time



   b) Start at 20, subtract 3 each time

   c) Start at –2, add 5 each time

   d) Start at 50, no change

**With your teacher.**

3. Write the first 5 terms ($t_1$, $t_2$, $t_3$, $t_4$, $t_5$) for each.

   a) You save \$25 every week, starting with \$0.

   b) A car loses \$1500 in value each year, starting at \$18 000.

   c) A plant grows 2 cm per day, starting at 5 cm.

   d) A leaky rainwater tank loses 10 litres of water every hour, starting at 120L



4. Find the common difference (d), then identify the 1st term ($t_1$) and the 3rd term ($t_3$).

| | | Common difference (d) | 1st term ($t_1$) | 3rd term ($t_3$) |
|---|---|---|---|---|
| a) | 12, 15, 18, 21, … | | | |
| b) | 40, 35, 30, 25, … | | | |
| c) | –5, –2, 1, 4, … | | | |
| d) | 7, 7, 7, 7, … | | | |
| e) | 2.5, 5, 7.5, 10, … | | | |
| f) | 100, 80, 60, 40, … | | | |

## Try it yourself (INTRO TO SEQUENCES)

1. State whether each of the following sequences are arithmetic. If they are, state the common difference (d), first term $(t_1)$ and the third term $(t_3)$
    a) 9, 12, 15, 18, …
    b) 16, 8, 4, 2, …
    c) 5, 3, 1, –1, –3 …
    d) 2, 5, 9, 14, …
    e) –10, –5, 0, 5, …
    f) 7, 7, 7, 7, …

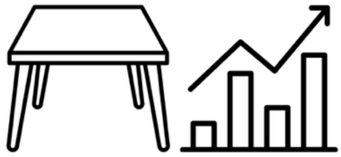
2. Fill in the missing terms.
    a) 2, __, __, 8, 10
    b) __, 14, 11, 8, __
    c) –3, __, 1, __, 5
    d) 50, 45, __, 35, __

3. Identify the first term $(t_1)$, then write the next 5 terms in each sequence (6 total).
    a) A phone plan costs $30 in the first month and increases by $2 per month.
    b) A tank loses 5 litres of water each hour, starting at 40 litres.
    c) A runner improves her time by 0.4 seconds each race, starting at 15.0 seconds.

4. Create your own sequence of 5 terms, clearly identifying the first term $(t_1)$ and common difference (d) in each.
    a) Write any arithmetic sequence that shows growth (increasing).
    b) Write any arithmetic sequence that shows decay (decreasing).
    c) Write an arithmetic sequence that includes a negative number.
    d) Write an arithmetic sequence whose first term is 0.
    e) Write a constant arithmetic sequence.

5. A sequence starts at 12 and every term is 7 less than the one before.
    a) Write the first 6 terms.
    b) Is this growth or decay?
    c) What is the common difference (d)?
    d) Will this sequence eventually become negative?
    e) What will the 10th term be?

## TABLES AND GRAPHS



We now know that an arithmetic sequence changes by adding or subtracting the same amount each time.

To understand them more clearly visually, we can show them in tables and graphs.

**Turning sequences into tables**

Make a column (or row) for each value you want to record.

Standard columns would be term position (n) and value of each term ($t_n$)

**Worked example. From sequence → table**

Turn the following sequence into a table: 4, 7, 10, 13, 16, …

We list the term position (n) and the value of the term ($t_n$):

| Term position (n) | Value ($t_n$) |
|---|---|
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |



**With your teacher.** Create a table for the following sequences.

a) 5, 9, 13, 17, 21, 25, 29, 33, 37

| Term position (n) | Value ($t_n$) |
|---|---|
| 1 | 5 |
| 2 | 9 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

b) 40, 36, 32, 28, 24, 20, 16, 12, 8

| Term position (n) | Value ($t_n$) |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

**Turning sequences into graphs**

1. Make a table of term position (n), term value ($t_n$), and points (n, $t_n$)
2. Plot the points (n, $t_n$)

**Worked example. From sequence → graph**

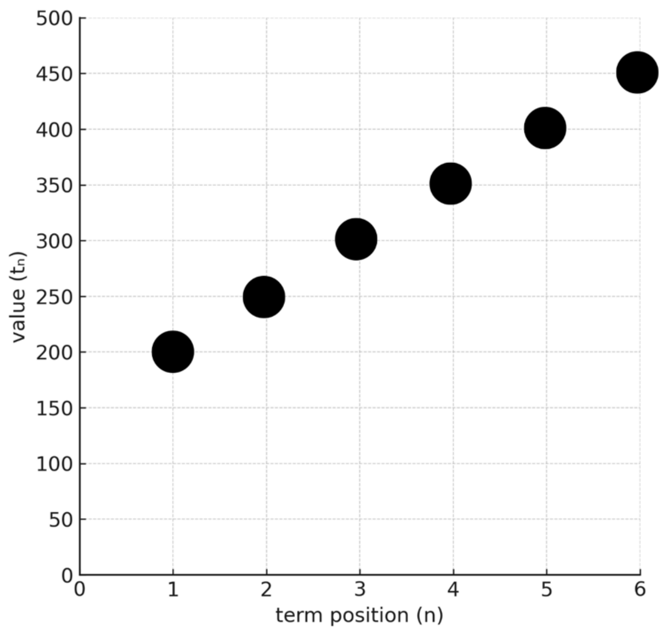
Jasmin starts a new job and earns $200 in her first week. Each week, she earns $50 more than the previous week as she takes on extra responsibilities.

This sequence could be written as: 200, 250, 300, 350, 400, 450, …

**Step 1:** Make a table of term position (n) and term value ($t_n$) for the first 6 terms.

| Term position (n) | Value ($t_n$) | | Point @ (n, $t_n$) |
|---|---|---|---|
| 1 | 200 | → | (1, 200) |
| 2 | 250 | → | (2, 250) |
| 3 | 300 | → | (3, 300) |
| 4 | 350 | → | (4, 350) |
| 5 | 400 | → | (5, 400) |
| 6 | 450 | → | (6, 450) |

**Step 2:** Plot the points. We only plot the points (discrete values). We do not join them with a continuous line because sequences only exist at set intervals.




When we plot arithmetic sequences on a graph, they form a straight-line pattern.

**With your teacher.**

A radioactive sample decays by 12 grams every hour. At the start of the monitoring period the sample has 100 grams of material remaining.

a) Represent this situation using a sequence. Write the first 6 terms ($t_1 \rightarrow t_6$)

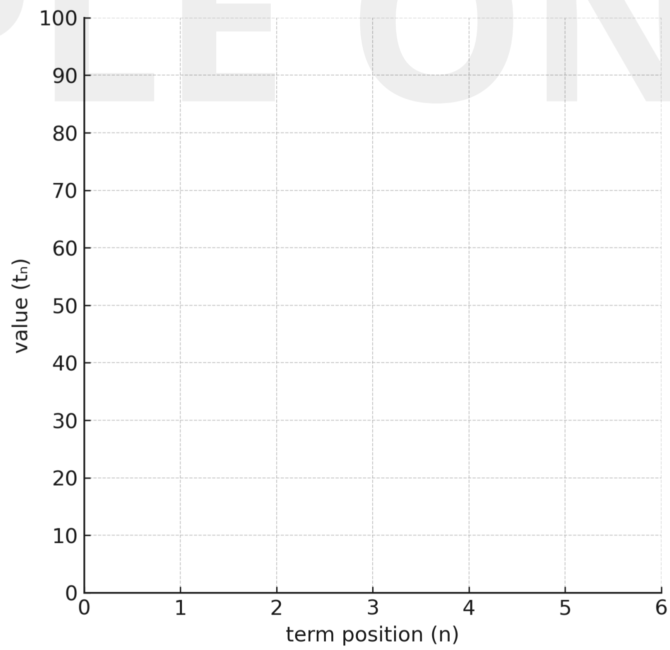
b) Identify the first term ($t_1$) and common difference (d).

c) Fill in the below table.

| **Term position (n)** | **Value ($t_n$)** | | **Point @ (n, $t_n$)** |
|---|---|---|---|
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |



d) Graph your points.



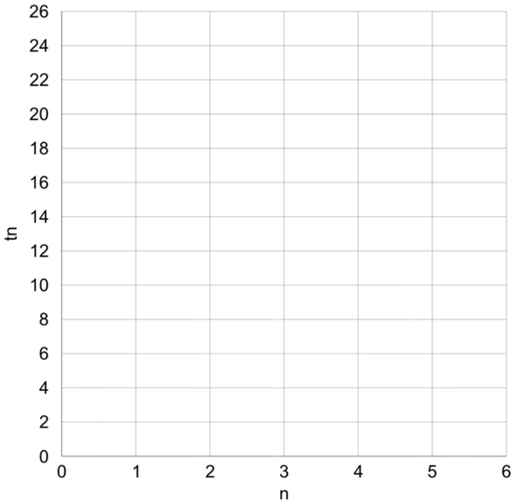

e) If we look at the 10th term, the sample has a weight of -8 grams. Is this possible in real life? Discuss.

## Try it yourself (TABLES AND GRAPHS - ARITHEMTIC)

1. Write each sequence as a table (n, $t_n$), then plot the first 6 terms of each sequence on a graph.

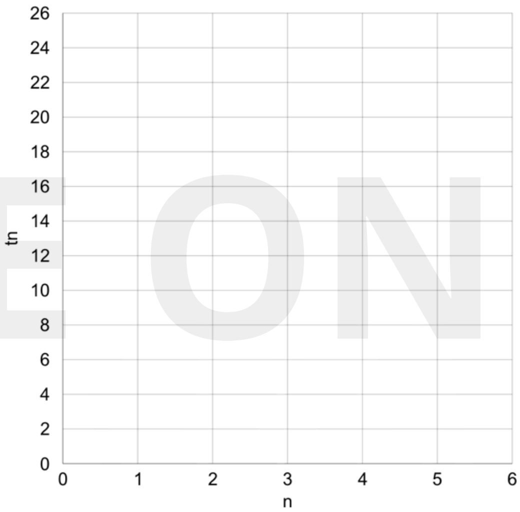
a) 5, 9, 13, …

| n | $t_n$ | | (n, $t_n$) |
|---|---|---|---|
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |




b) 12, 10, 8, …

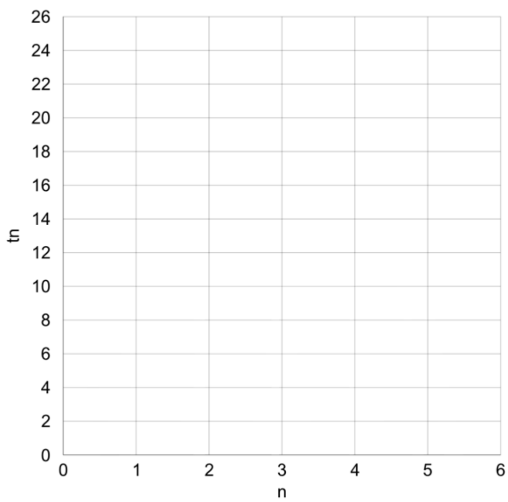
| n | $t_n$ | | (n, $t_n$) |
|---|---|---|---|
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |






c) 4, 4, 4, …

| n | $t_n$ | | (n, $t_n$) |
|---|---|---|---|
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |

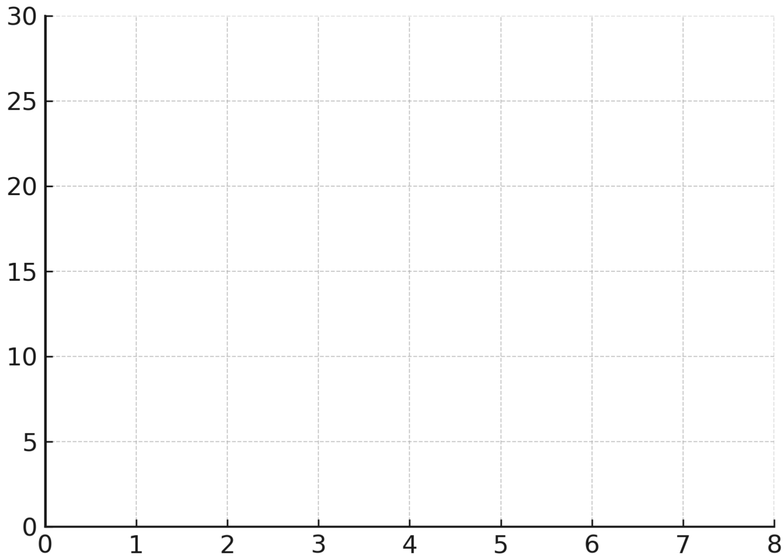



2. Answer the following questions for each graph in question 1:
   a) Is it increasing (growth), decreasing (decay), or constant?
   b) Does it form a straight-line pattern?
   c) What is the common difference (d)?

## Try it yourself (TABLES AND GRAPHS - ARITHMETIC cont)

3. For the sequence: 7, 10, 13, 16, 19, …
   a) Identify the common difference (d)
   b) Fill in the details in the table below and plot the first 8 terms.

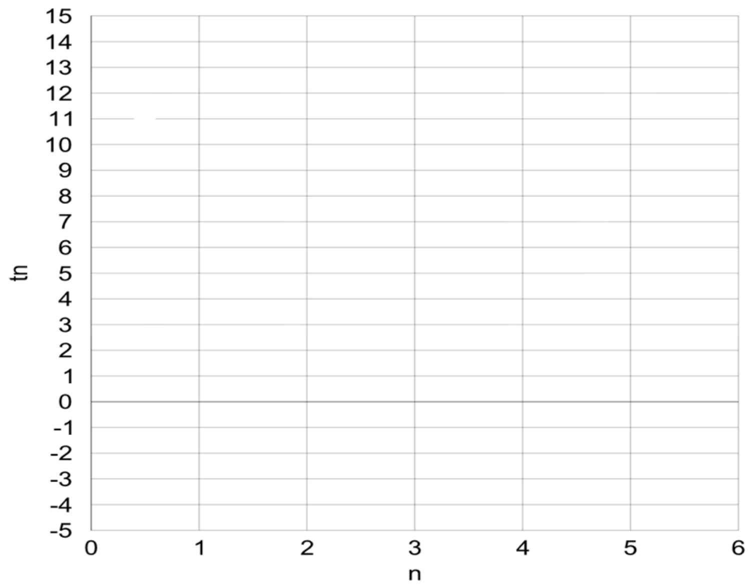
| n | $t_n$ | | (n, $t_n$) |
|---|---|---|---|
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |






4. A sequence has $t_1$ = 15 and d = –4.
   a) Fill in the table with the first 6 terms.
   b) Graph the first 6 terms.
   c) Does the graph show growth or decay?

| n | $t_n$ | | (n, $t_n$) |
|---|---|---|---|
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |
| | | → | |




5. True or False? Explain.
   a) All arithmetic sequences go up.
   b) An arithmetic sequence can have negative numbers.
   c) Arithmetic sequences always make a straight-line pattern on a graph.

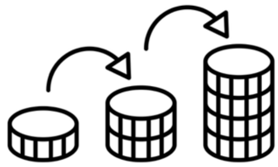
2027

# GENERAL MATHEMATICS

## Level 3

# FINANCE

## INTRODUCTION TO COMPOUNDING INTEREST



Compound interest means that the interest is added to the principal after each period. The principal, having been increased will earn more interest during the second interest bearing period than during the first. This second interest instalment is also added to the principal, increasing it again - and the cycle continues.

**Worked example.**

Alice deposits $1,000 in an account paying 6% pa compound interest calculated annually. How much will be in the account at the end of 3 years?

The problem can be solved by using the simple interest formula:

Interest for 1st year $I = P\,r\,t$

$I = 1000 \times 0.06 \times 1$

$I = \$60$

Amount in account after first year is $1,000 + 60 = $1,060

---

Interest for 2nd year $I = P\,r\,t$

$I = 1060 \times 0.06 \times 1$

$I = \$63.60$

Amount in account after second year is $1,060 + $63.60 = $1,123.60

---

Interest for 3rd year $I = P\,r\,t$

$I = 1123.60 \times 0.06 \times 1$

$I = \$67.41$

Amount in account after third year is $1,123.60 + 67.41 = $1,191.02

---



There is a compound interest formula that is useful to calculate the balance at any time. Before we jump into the formula, it’s important to understand how balances change step by step using recurrence relations.

**What is a Recurrence Relation?**

A recurrence relation is an equation that defines each new term of a sequence in terms of the previous term. In finance, it models how the balance of a loan or investment changes from one compounding period to the next.

***You may also see recurrence relations referred to as Difference Equations***

**Recurrence relation formula**

$$t_{n+1} = rt_n + d$$

Where:

( $r$ ): Rate of growth $(1+i)$ or decay $(1-i)$

( $i$ ): Interest rate per period

( $t_n$ ): Balance after ( $n$ ) compounding periods

( $d$ ): Deposit amount each period (+ for deposit, - for withdrawal/repayment)

**Worked examples.**

1. You deposit $1,000 into a savings account that earns 6% annual interest, compounded annually. Write the recurrence relation (or difference equation) and calculate the balance after 3 years.

   The recurrence relation is:

   $t_{n+1} = 1.06 \times t_n \qquad t_0 = \$1{,}000$

   Step-by-step calculation:

   Year 1: $t_1 = (1.06) \times 1{,}000 = 1{,}060$

   Year 2: $t_2 = (1.06) \times 1{,}060 \quad = 1{,}123.60$

   Year 3: $t_3 = (1.06) \times 1{,}123.60 = \mathbf{1{,}191.02}$ ← The balance after 3 years is $1,191.02



2. A loan of $10,000 has 12% annual interest, with $500 monthly repayments. Write the difference equation and calculate the balance for the first 3 years.

   The recurrence relation is:

   $$t_{n+1} = \left(1 + \frac{0.12}{12}\right) \times t_n - 500, \qquad t_0 = \$10{,}000$$

   Step-by-step calculation:

   Month 1: $t_1 = (1.01) \times 10{,}000 - 500 = 9{,}600.00$

   Month 2: $t_2 = (1.01) \times 9{,}600 - 500 = 9{,}196.00$

   Month 3: $t_3 = (1.01) \times 9{,}196 - 500 = 8{,}787.96$

| Starting balance | Interest added | Repayment | Closing balance |
|---|---|---|---|
| $10,000 | $100.00 | $500 | $9,600.00 |
| $9,600 | $96.00 | $500 | $9,196.00 |
| $9,196 | $91.96 | $500 | $8,787.96 |

*Table format*

**Key Takeaways of recurrence relations:**

- Recurrence relations let you calculate balances period by period.
- They help you understand how compound interest accumulates over time or how loans are gradually paid off.
- This approach is the foundation for the compound interest formula, which we will learn next.

**Complete with your teacher:**

1. An investment of $2,500 earns 5% annually. Write the recurrence relation and calculate the balance after 5 years.



2. A loan of $5,000 has 1% monthly interest and $200 monthly repayments. Write the recurrence relation and calculate the balance after 3 months.

3. Complete a table showing the balance of a $1,200 investment earning 8% annually for 4 years.

**Try it yourself (RECURRENCE RELATIONS).**

1. A deposit of $1,500 earns 7% annual interest, compounded annually.
   a) Write the recurrence relation.
   b) Calculate the interest for each of the first 4 years.
   c) How much interest was earned in total after 4 years?

2. A loan of $8,000 has a monthly interest rate of 0.9% with repayments of $300 each month.
   a) Write the recurrence relation.
   b) Calculate the balance after 1, 2, and 3 months.
   c) Explain how the repayments affect the remaining balance over time.

3. You invest $600 at 10% annual interest.
   a) Write the recurrence relation.
   b) Calculate the balance for each of the first 3 years.

4. $3,000 is deposited into an account earning 4% interest annually. At the end of each year, an additional $500 is deposited.
   a) Write a recurrence relation that includes the yearly deposit.
   b) Calculate the balance for the first 4 years.



5. Account A: $2,000 invested at 6% annual interest. Account B: $2,000 invested at 6% annual interest with an additional $200 added at the end of each year.
   a) Write the recurrence relations for both accounts.
   b) Create a table for each account (like the one in worked example 2), showing the balances and interest added each year over the course of 5 years.
   c) How much of a difference did the additional $200 make over the 5 years?

6. You take out a loan of $10,000 with an interest rate of 20%p.a. compounding monthly, and monthly repayments of $800. Write the difference equation and calculate the balances for the first 6 months.

## THE COMPOUND INTEREST FORMULA

Now that you understand recurrence relations and how balances can change period by period, we will explore a faster way to calculate the same results: the compound interest formula.

**Compound interest formula**

$$\boldsymbol{FV = PV(1+i)^n}$$

Where:

FV = Future Value

PV = Present Value

i = Interest rate per period (as a decimal or fraction)

n = Number of interest bearing periods

In the previous section, we used recurrence relations to calculate how money grows or how a loan balance changes over time. Now, let's see how we can use a formula to get the same result more quickly. We'll compare both methods side by side.



**Worked example: Comparing methods**

You invest **$2,000** at **5% annual interest,** compounded annually for **5 years.**

| Recurrence relations: | Compound formula: |
|---|---|
| Year 1: 1.05 x $2,000 = $2,100 | |
| Year 2: 1.05 x $2,100 = $2,205 | $FV = PV(1+i)^n$ |
| Year 3: 1.05 x $2,205 = $2,315. 25 | $FV = \$2{,}000(1+0.05)^5$ |
| Year 4: 1.05 x $2,315.25 = $2,431.01 | $FV = \$2{,}552.56$ |
| Year 5: 1.05 x $2,431.01 = $2,552.56 | |

**Comparison:**

- Both methods give the **same result.**
- Recurrence relations calculate balances **period by period,** which is helpful for understanding how compound interest works.
- The formula is a **shortcut** for quickly finding the future value over many periods, imagine doing recurrence relations for 30 years!

**Worked Examples.**

1. You deposit $1,000 at 6% annual interest, compounded annually for 3 years. Find the future value after 3 years.

$$FV = \$1{,}000(1 + 0.06)^3$$
$$FV = \$1{,}191.02$$

2. An investment of $5,000 earns 8% annual interest, compounded quarterly for 2 years. Find the value after 2 years.

$$FV = \$5{,}000\left(1 + \frac{0.08}{4}\right)^{2\times 4}$$
$$FV = \$5{,}000(1 + 0.02)^8$$
$$FV = \$5{,}858.30$$

**With your teacher:**

1. An investment of $2,000 earns 5% annual interest, compounded annually for 6 years. Find the future value.



2. A loan of $10,000 has 7% annual interest, compounded monthly. What is the amount owed after 5 years if no payments are made?

**Try it yourself (COMPOUND INTEREST FORMULA).**

1. Explain why using the formula is more practical for long-term investments.

2. Use the compound interest formula to find the future value of the following:
    a) \$8,500 invested at 3%p.a. compounding monthly for 3 years.
    b) 2.5%p.a. interest rate, compounding weekly for 2 years. \$9,000 invested.
    c) Quarterly compounding, 10% annual interest, 5 years, \$7,560 invested.
    d) \$200 invested at 6%p.a. compounding fortnightly for 10 years
    e) \$500 invested for 60 years. Interest rate of 4% p.a. compounding yearly.

3. A deposit of \$1,200 earns 10% annual interest, compounded quarterly for 3 years. Find the future value.

4. An investment of \$1,500 earns 6% annual compounding interest for 3 years. Use both recurrence relations and the compounding formula to find the future value.

5. \$3,000 is invested at 8% annual interest, compounded annually for 5 years. Compare results using both methods (recurrence and formula)..

6. A loan of \$10,000 has an interest rate of 7%p.a. compounding monthly. Calculate the amount owed after 2 years with both methods (recurrence and formula).



7. Calculate the future value for each of the below after 5 years and decide which is better. Explain why the values differ despite having the same interest rate.
    Investment A: \$1,500 at 8% annual interest, compounded annually.
    Investment B: \$1,500 at 8% annual interest, compounded quarterly.
    Investment C: \$1,500 at 8% annual interest, compounded monthly.
    Investment D: \$1,500 at 8% annual interest, compounded daily.

8. Mia wants to save for a big overseas holiday. She deposits \$4,000 into a high-interest savings account that offers 4.5% annual interest, compounded annually. Mia plans to leave the money untouched for 5 years. How much money will Mia have saved by the time she's ready to book her trip?

9. Alex invests \$7,500 for 4 years. Interest is added half-yearly. For the first two years the rate per annum is 8 % and for the remaining 2 years, the rate per annum is 10 %. How much does he have in the end?

## SOLVING FOR VARIABLES IN THE COMPOUND INTEREST FORMULA

We have learned how to calculate the future value (FV) of investments and loans using the compound interest formula. In this section, we will go further and solve different types of problems where we might need to find:

- The Present Value (PV)
- The Interest Rate (i)
- The Number of Compounding Periods (n)

All of these can be solved using rearranged formulas. Solving for interest rate or number of periods can get complicated – it is recommended to use your CAS calculator.

**CAS Calculator – finance solver**

**Menu $\longrightarrow$ Financial $\rightarrow$ compound interest**

N = Number of payment (instalment) periods

I% = Annual interest rate (as a %)

PV = Present value

PMT = Amount paid each period

FV = Future value

P/Y = Payments (instalments) per year

C/Y = Compounding periods per year

***Notes:***

- In format – ensure payments are set to 'END' (end of period).
- Use positive and negative signs to show money coming into or going out of your pocket.

  *Money paid out $\rightarrow$ negative*

  *Money received $\rightarrow$ positive*



**Worked example.**

Moya wants to withdraw $15,000 in 7 years from an account that earns 3.5% p.a. compounding monthly. How much should she invest now to make that happen?

*Enter the values on the right into the finance solver. Then select PV and press SOLVE. Remember that the sign represents money coming into or going out of your pocket.*

***PV = –$11,744.76***

***Moya must put $11,744.76 in the account now.***

N = 7 $\times$ 12
I% = 3.5
PV = SOLVE
PMT = 0
FV = 15000
P/Y = 12
C/Y = 12
END

## Finding Present Value (PV) – rearranged formula

$$PV = \frac{FV}{(1+i)^n}$$

**Worked examples.**

1. You want $10,000 in 5 years and can earn 6% p.a. compounding annually. How much should you invest now?

**Using rearranged formula:**

$$PV = \frac{\$10{,}000}{(1+0.06)^5}$$

$$PV = \$7{,}472.58$$

*Need to invest $7,472.58 today.*

**Using CAS calculator:**

N = 5

I% = 6

PV = SOLVE

PMT = 0

FV = 10000

P/Y & C/Y = 1

END



2. Irene invests some money and is told it will be worth $25,000 in 10 years. She is going to earn an interest rate of 4.5% p.a., compounding weekly. How much did she invest?

**Using rearranged formula:**

$$PV = \frac{\$25{,}000}{(1+\frac{0.045}{52})^{520}}$$

$$PV = \$15{,}943.81$$

*She invested $15,943.81*

**Using CAS calculator:**

N = 10 x 52

I% = 4.5

PV = SOLVE

PMT = 0

FV = 25000

P/Y & C/Y = 52

END

**With your teacher.**

How much needs to be invested now to have $12,000 in 10 years at 6% p.a. compounding fortnightly?

**Worked examples.**

**1.** You invest $2,000 and after 4 years have $2,400. Find the interest rate if the investment compounds annually.

**Using CAS calculator:**

N = 4

I% = SOLVE → **4.66%**

PV = 2000

PMT = 0

FV = -2400

P/Y = 1

C/Y = 1

END

**2.** You invest $3,000 at 5%p.a. compounding annually. How many years until it doubles?

**Using CAS calculator:**

N = SOLVE → **14.2 years**

I% = 5%

PV = 3000

PMT = 0

FV = -6000

P/Y = 1

C/Y = 1

END

**With your teacher.**

1. An investment of $2,500 grows to $3,000 in 3 years. Find the annual interest rate if the investment compounds monthly.



2. How many years will it take for $4,000 to grow to $8,000 at 7%p.a. compounding monthly?

3. How long will it take for $10,000 to reach $15,000 at 4%p.a. compounding fortnightly?

**Try it yourself (SOLVING FOR VARIABLES - COMPOUNDING).**

1. Find the present value of the following investments:
   a) $2,000 in 5 years with 4% interest compounding monthly.
   b) $10,000 received in 6 years with 6%p.a. compounding fortnightly.
   c) 3% p.a. compounding daily (365 days) for 10 years, $52,500 final amount.

2. Sarah wants $3,500 in her account in 10 years. If she makes one deposit today and leaves it untouched at 5% p.a. compounding yearly, how much does she need now?

3. A company needs $50,000 in 10 years for equipment replacement. If the interest rate is 7%p.a. compounding monthly, how much should they set aside today?

4. Tom wants to buy a house in 12 years and estimates it will cost $400,000. How much should he invest now at 5% annual interest to reach this goal?

5. Calculate the annual interest rate for the following: (round to 2 decimal places)
   a) $2,000 becomes $2,500 after 4 years. Compounds weekly.
   b) $1,000 grows to $1,100 in 2 years. Compounds fortnightly.
   c) A savings account that compounds monthly triples in value over 15 years.



6. Tim wants to know what interest rate he would need to get if he wants his $50,000 house deposit to grow to a $120,000 in the next 5 years. Interest compounds monthly and he doesn't plan on making any additions to the $50k.

7. Calculate how many years the following amounts were invested:
   a) $4,000 reaches $10,000 after being invested at 8%p.a. compounding weekly.
   b) Retirement fund grows from $50,000 to $150,000 at 7% compounding yearly.
   c) $3,000 doubles when invested at 12% p.a. compounding daily (365 days).

8. Jane is trying to figure out when she can retire. She has a lump sum of $1,000,000 and estimates she will need $1,200,000 to retire comfortably. She can get a deal that offers 8% p.a. compounding monthly. How long until she can retire?

**Try it yourself (MIXED COMPOUND INTEREST PROBLEMS).**

1. $ 1500 is invested at 6 % pa interest compounding annually. Find the amount in the account after 6 years.

2. Find the total in an account if:
   a) $3,000 is invested at 8% pa interest compounding quarterly for 5 years.
   b) $1,250 is invested at 9% pa interest compounding monthly for 7 years.
   c) $3,428 is invested at 4% pa interest compounding quarterly for 6 years.
   d) $2,500 is invested at 5.5% pa interest compounding monthly for 5 years.
   e) $1,600 is invested at 6.25% pa compounding twice per year for 4.5 years.
   f) $7,250 is invested at 5.8% pa interest compounding quarterly for 5 ½ years.

3. $4,500 is invested at 6% pa compounding quarterly.
   a) Find the amount in the account after 6 years.
   b) Find the amount of interest earned.

4. $5,250 is invested at 5.5 % pa compounding quarterly.
   a) Find the amount in the account after 4 years.
   b) Find the amount of interest earned.



5. A $7,500 investment is placed in a term deposit for 4 years. Interest is compounded half-yearly. The investment earns 8% p.a. for the first 2 years and 10% p.a. for the final 2 years. Determine the value of the investment at the end of the 4-year term.

6. Jacqui wishes to invest $4,000 for 6 years. Which of the following provides the best investment opportunities? (In each case, give the final amount that would be returned to the investor):
   a) interest compounded annually at 7%
   b) interest compounded half yearly at a rate/annum of 7%
   c) interest compounded monthly at a rate/annum of 7%.

7. A man wishes to invest $5,000 for 10 years. Find his best investment alternative given the following choices:
   a) compound interest at 7% pa adjusted annually;
   b) compound interest adjusted monthly at a rate of 6% pa.

**Try it yourself (MIXED COMPOUND INTEREST PROBLEMS).**

8. In your own words explain why an investment which offers 6% pa interest compounded monthly will be better than one which offers 6% pa interest compounded annually.

9. Draw a graph which shows the growth of a $1,000 investment at 9% pa interest compounding monthly over 20 years. Explain why the graph has its shape.

10. Calculate the principal required to achieve $20,000 in 10 years if interest is added quarterly and the annual rate is 6%.

11. Calculate the principal required to achieve $5,000 in 6 years if interest is added monthly and the annual rate is 6.75%.

12. Jill needs $7,000 for a trip in 3 year's time. Her credit union offers a rate per annum of 5% which will be compounded quarterly. How much should she deposit (to the nearest dollar)?

13. Now use your calculator's financial mode to recalculate some of the problems in this set as directed by your teacher.



14. Use your calculator's financial mode to find:
    a) The interest rate required for $1,000 to increase to $1,500 in 5 years with interest added annually.
    b) The interest rate required for $2,500 to increase to $3,000 in 5 years with interest added quarterly.
    c) The interest rate required for $5,000 to increase to $8,000 in 10 years with interest added monthly.
    d) The time required for $1,000 to increase to $1,500 at 5% compound interest pa added annually.
    e) The time required for $3,500 to increase to $4,000 at 8% compound interest pa added monthly.
    f) The time required for $6,000 to increase to $7,000 at 6% compound interest pa added quarterly.

2027

# GENERAL MATHEMATICS

## Level 3

# NETWORKS

**Maximum flow by inspection.**

In a flow network, each edge has a capacity which tells us the maximum amount that can pass through it.

To find the maximum flow from the source to the sink, we can sometimes use logic alone — this is called finding the maximum flow by inspection.

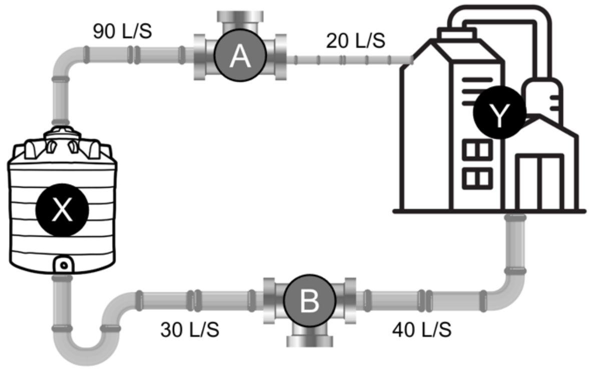
The total flow through a network is limited by the smallest capacity on each possible route from the source to the sink. This edge acts like a *bottleneck* — it limits how much can flow through that entire path.

If there are several possible paths, the overall maximum flow is the sum of the flows through each path, as long as they don't share a bottleneck edge.

| Steps to find maximum flow (by inspection) | |
|---|---|
| **Step 1:** | **Identify the source and sink.**<br>These are the start and end points of the flow. |
| **Step 2:** | **List all possible paths** from source to sink. |
| **Step 3:** | **Find the bottleneck** for each path<br>— the smallest capacity (weight) along that path. |
| **Step 4:** | **Add the flows** from each path together, provided the paths don't overlap.<br>If paths share an edge, that edge's total flow can't exceed its capacity. |



**Worked example.**




A water tank **(X)** pumps water to a treatment plant **(Y)** through two routes, A and B.

Each pipe has a maximum capacity, measured in litres of water per second (L/S).

X is the source as all pipes lead from it.

Y is the sink as all pipes lead to it.

Possible paths are path A (X → A → Y) and path B (X → B → Y)

The water tank can pump a maximum of 20 L/S through path A at a time.

The water tank can pump a maximum of 30 L/S through path B at a time.

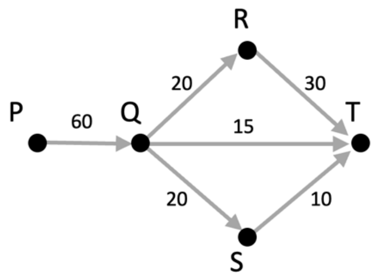
So, using both paths, the **total maximum flow from X to Y is 50 L/S.**

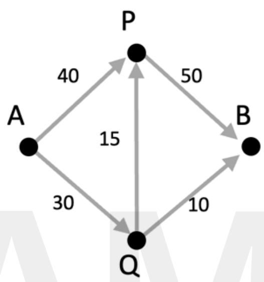
If we wanted to increase the total flow through the system, we would need to upgrade the smallest-capacity pipes — the ones that are restricting flow along each path.

**With your teacher.**

Identify the source and sink in each of the following networks.

Find 'by inspection' the maximum flow from source to sink.






Finding the maximum flow *by inspection* works fine for small networks.

But most real-life networks are not small – like planning water pipes for towns, motor vehicle traffic through roads and highways, or running data cables to reach homes.

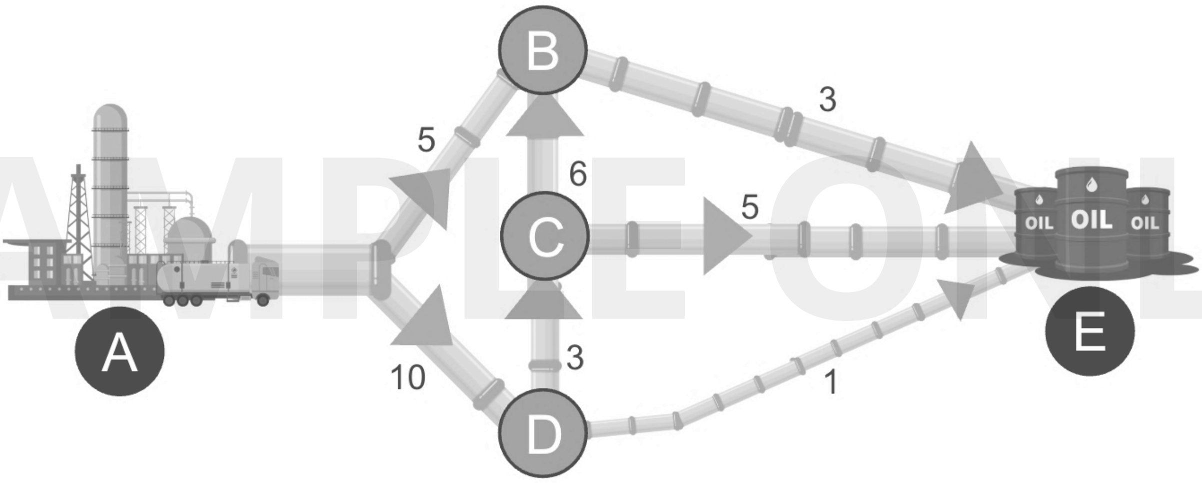
When the networks get larger, the **Minimum cut – Maximum flow theorem** gives us proof that we've reached the true maximum.

| **Minimum cut – Maximum flow theorem** |
|---|
| In any flow network, the **maximum possible flow from the source to the sink is equal to the total capacity of the minimum cut** — the smallest total capacity of edges that, if removed, would completely separate the source from the sink. |

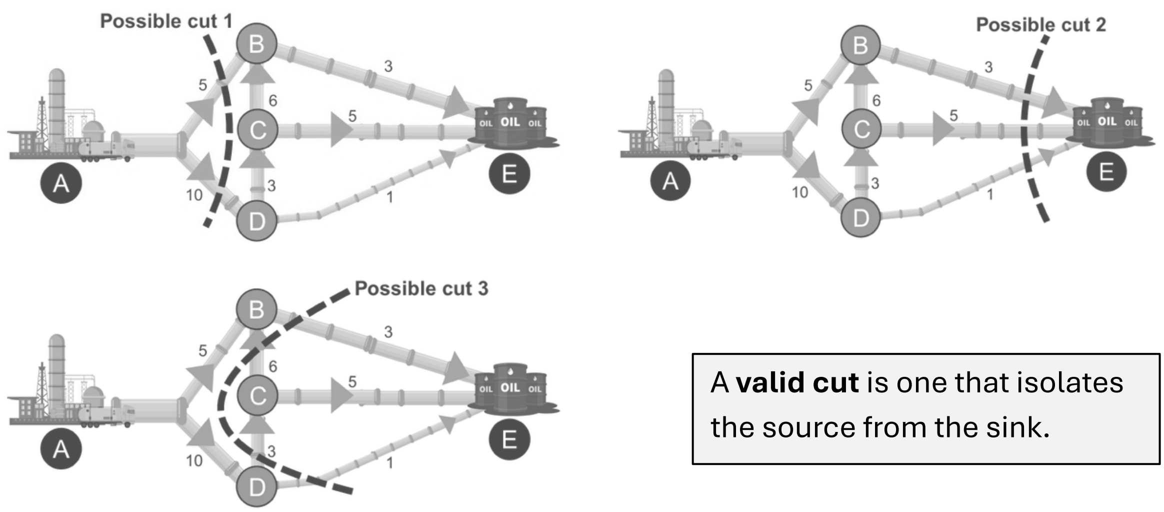
The *weakest link* in the network — the narrowest set of pipes connecting the source side to the sink side — determines how much flow the whole system can carry.

| Steps to minimum cut – maximum flow theorem. |
| --- |
| 1. Cut the flow network in two – completely separating the source from the sink.<br>2. The **total capacity** of that cut tells you how much flow would be blocked by cutting those edges.<br>3. There are often multiple possible cuts. The cut with the smallest possible capacity (the **minimum cut**) results in the **maximum possible flow** that the network could carry. |

**Worked example.**
Determine the maximum volume of oil that can flow through the network of pipes from an oil processing plant (A) to storage (E). Measurements in litres per second.




**Step 1:** Find all the places where we could cut the network that would completely stop the flow of oil from the source (A) to the sink (E).



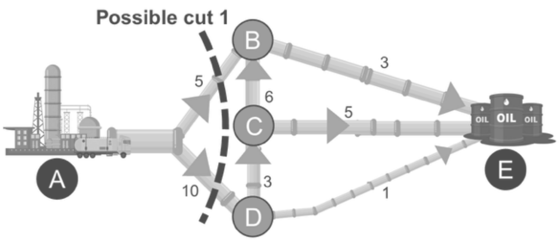

A **valid cut** is one that isolates the source from the sink.

**Worked example continued.**

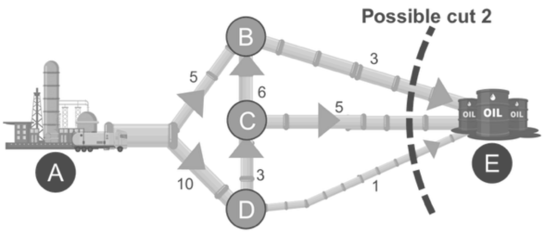
**Step 2.** Find the capacity of each cut.

(how much oil would be blocked if that cut were made)

Note: Not all edges are counted, only the ones that would stop the flow of oil.

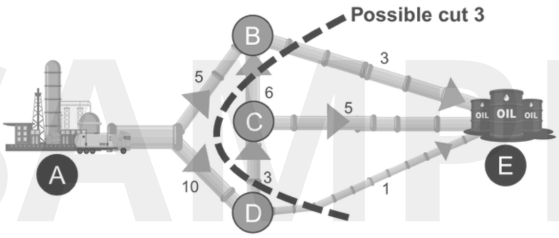



Cut through A → D = 10
Cut through A → B = 5
**Total capacity of cut 1 = 15**




Cut through B → E = 3
Cut through C → E = 5
Cut through D → E = 1
**Total capacity of cut 2 = 9**

Cuts are only counted if they stop the flow from the source to the sink.




Cut through B → E = 3
Cut through D → C = 3
Cut through C → B = 0 (the oil was already cut off at D → C)
Cut through D → E = 1
**Total capacity of cut 3 = 7** ← minimum cut



The **minimum cut** (cut #3) represents the **weakest choke point in the whole system** — the *smallest total capacity* of pipes that, if blocked, would completely stop oil from getting from A (source) to E (sink).

- If you were an engineer trying to increase total flow, the pipes involved in cut #3 are the pipes you should upgrade.
- If you were trying to shut the system off, the pipes in cut #3 are the fewest pipes you would need to close to completely stop all oil reaching E.

**What the minimum cut tells us.**

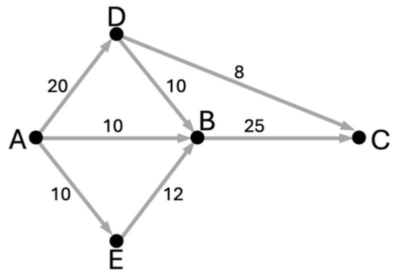
7 litres per second is the true maximum possible flow from A to E — the entire network cannot deliver more than that.

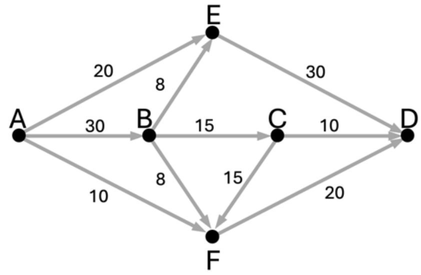
The edges on that minimum cut are the weakest links — they're the ones limiting how much oil can flow through the whole system.

This example is based on the example shown in a YouTube video by *Juddy Productions* called "*Minimum cuts and maximum flow rate*". Check it out if you are having trouble with this concept

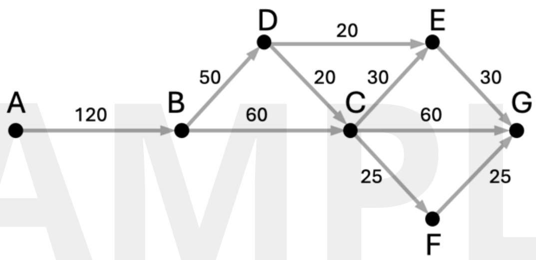
**With your teacher.**

1. Find the maximum flow from source to sink in the following networks.




2. A highway network is represented by the graph below. Weights represent the capacity of different sections of the highway in cars per minute.
   a) Find the maximum flow of traffic from A to G.

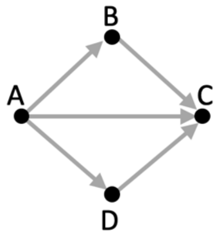


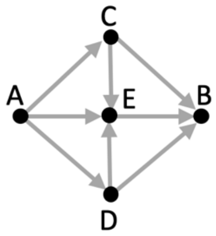

   b) Authorities would like to improve traffic flow but wish to improve only one section of road. Which sections will bring about improvement?

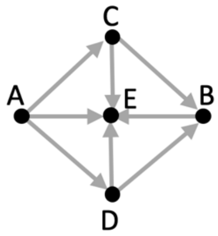
   c) By how much would the overall flow improve if the section DE was to be upgraded to 60 cars/min?

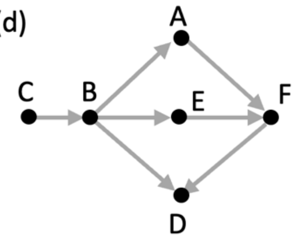
## Try it yourself (FLOW)

1. Identify the source and the sink in the following networks.

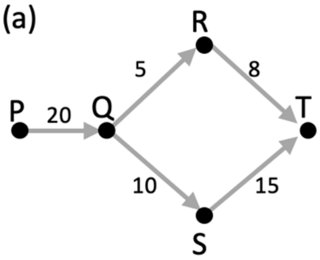
(a)


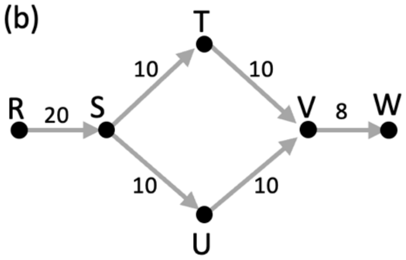

(b)


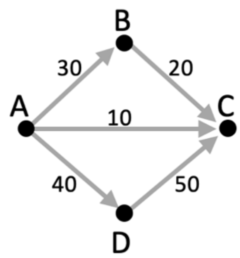

(c)


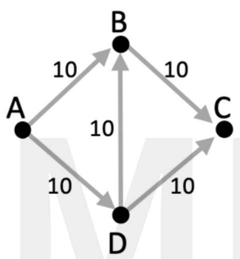

(d)


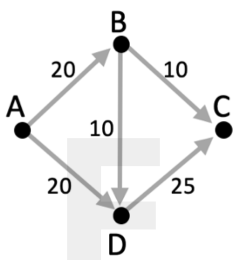

2. Find the maximum flow in each of the following networks 'by inspection'

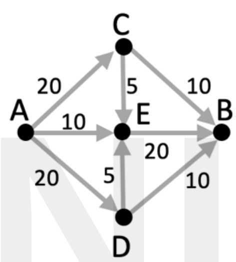
(a)



(b)


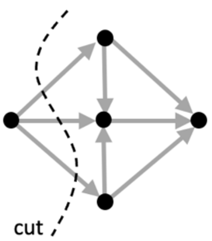

(c)


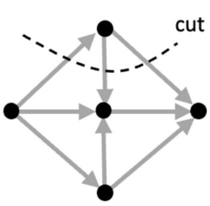

(d)


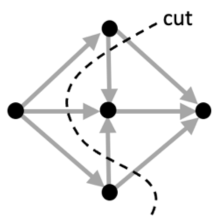

(e)


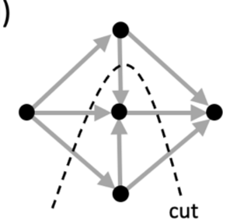

(f)



3. A network cut is valid if it separates source and sink. Which of the following network cuts are valid?

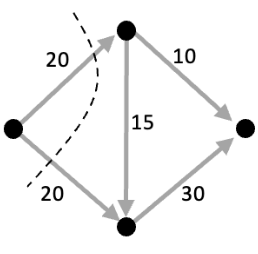
(a)


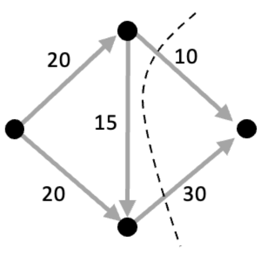

(b)


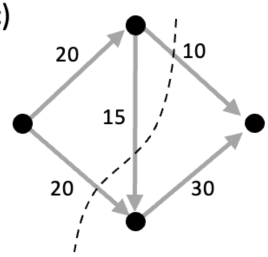

(c)


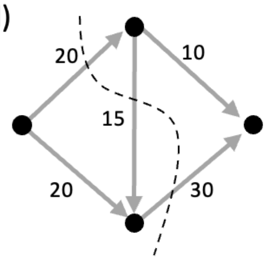

(d)


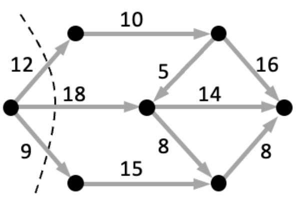

4. Calculate the capacity of each of the following network cuts.

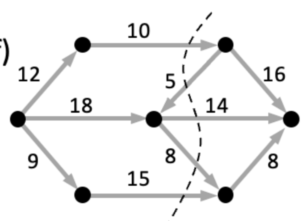
(a)



(b)



(c)



(d)


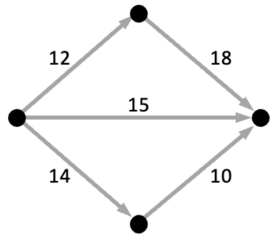

(e)



(f)


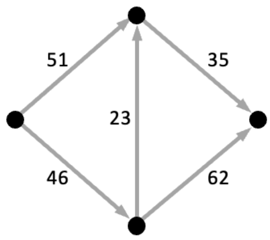
**Try it yourself (FLOW cont)**

5. Use ‘min cut = max flow’ to calculate the maximum flow of each of the following networks.

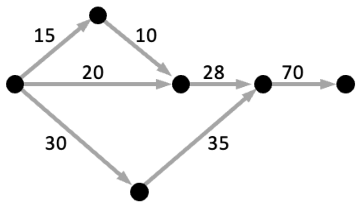
(a)



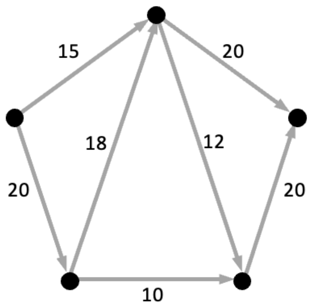

(b)




(c)




(e)

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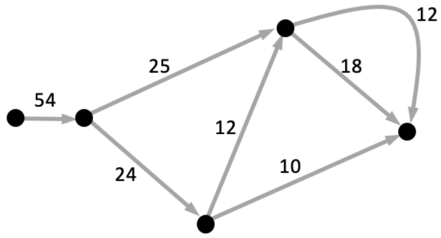
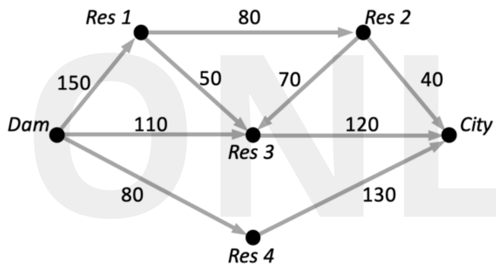
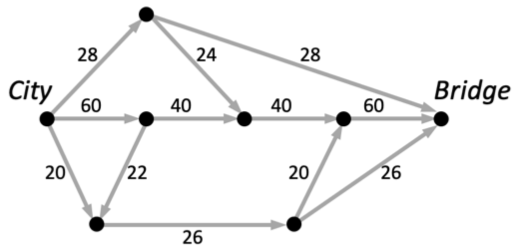



6. A network of pipes and reservoirs reticulate fresh water to a small city. The weights on the graph give the capacities of the pipes in ML/h. What is the maximum flow of water to the city?

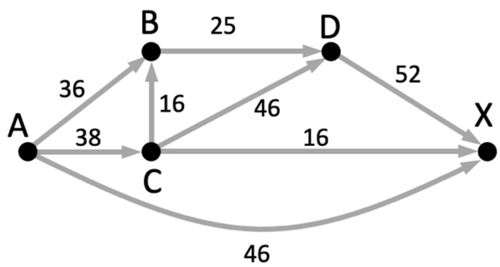





7. The graph presents the major roads between the centre of a city and the city’s bridge. Weights detail the capacities of each road in cars per minute. What is the maximum flow of the network?

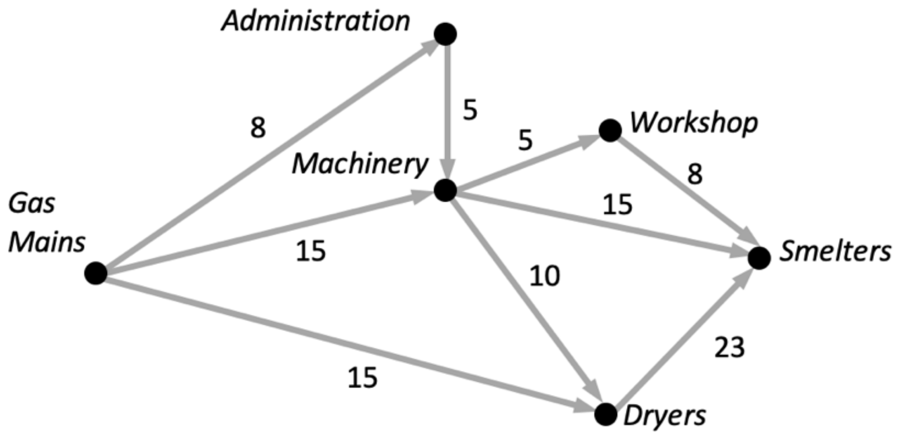



8. The graph shows the number of tonnes of air-freight that can be sent every day via different routes between two capital cities.
   a) What is the maximum flow of air freight from city A to city X?
   b) How could the overall capacity be increased by increasing the freight capacity on a single route. List some alternatives.

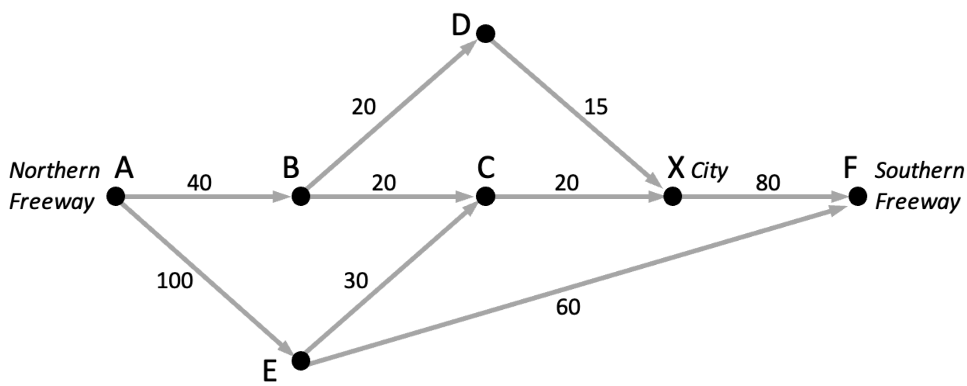


**Try it yourself (FLOW cont)**

9. Natural gas is pumped from a gas mains to different parts of a large factory according to the graph. Most gas is used in the smelters.
   a) What is the maximum capacity of the network to supply the smelters?
   b) In order to supply more gas to the smelters it is planned to upgrade the line between the mains and 'Machinery'. What is the maximum worthwhile upgade to this line?

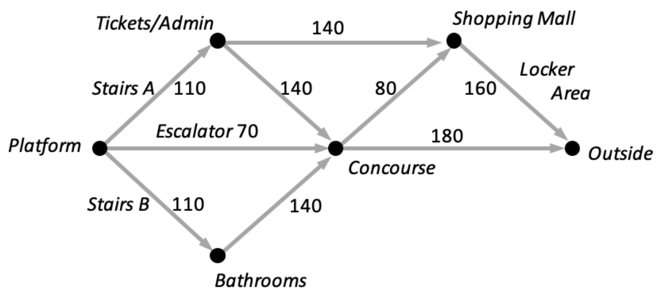



10. The graph shows how freeway traffic flows through and around a small city. (Weights are measured in cars per minute)
    a) Find maximum flow across the network from freeway to freeway.
    b) Which roads (considered individually) could be upgraded to improve flow?
    c) The city council is considering upgrading the road from D to X. What would be the maximum worthwhile upgrade and what would be the improved capacity of the system in this case?
    d) If, instead, the council was to upgrade the road from E to F; what would be the maximum worthwhile upgrade and what would be the improved capacity of the system?



**Try it yourself (FLOW cont)**

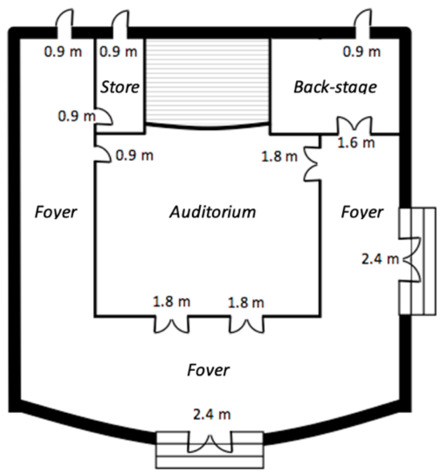
11. The graph shows the capacity of routes to exit a subway train station. (Weights are measured in people per minute). Authorities wish to upgrade the capacity of the station to deal with an increasing number of passengers.
    a) What is the maximum flow of people from the platform to the 'outside'.
    b) Which edges on the graph (considered singularly) would, if upgraded, result in an overall improvement of flow?




    c) Find the maximum worthwhile upgrade to the escalator and the resulting maximum flow of the network.
    d) It has been suggested that moving the lockers to the corridor between the concourse and the shopping mall would improve the flow of their current corridor by 20 (ppm) but decrease the flow in their new position by 20 (ppm). Would this be worthwhile given that the escalator has been improved by your recommended amount?



12. The rate at which people can exit a building depends upon the width of its doors. The capacity of a door with regard to the throughput of people can be found using the formula: $C = 40\ w^2$ (people per minute) where w is the width of the door in m.
    a) Find the capacity of each of the doors of the building in the diagram.
    b) Draw an access network for the building weighted with the capacity of each door.




    c) Find the maximum flow of people in the event of an evacuation.
    d) How long would it take to evacuate if the auditorium holds 2500 people?
    e) Which of the following renovations would speed the time of an evacuation?
        - Make the foyer larger in size.
        - Widen the front and side external entrance doors.
        - An additional set of doors joining the auditorium to the foyer.
        - Make the door from 'back-stage' to outside a double door.

2027

# GENERAL MATHEMATICS

## Level 3

# TRIGONOMETRY

Unit 4B
TRIG

# PART 1: RIGHT-ANGLED TRIANGLES

Trigonometry is the branch of mathematics that deals with triangles. The word comes from "trigon" (triangle) and "metron" (measure) in Greek, and translates to "measuring triangles".

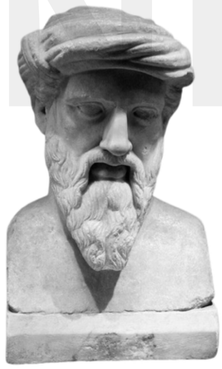
Trigonometry allows us to solve real-life problems like finding heights of buildings, lengths of ramps, and distances across rivers. It is frequently used in navigation, construction, and engineering.

In this trigonometry unit we will cover:

- Using Pythagoras' theorem
- Using trigonometric ratios (SOH CAH TOA)
- Finding the area of a triangle
- Using angles of elevation and depression



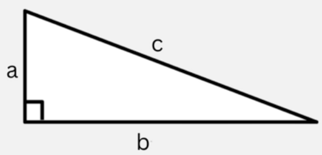
## PYTHAGORAS' THEOREM



Pythagoras of Samos was a very interesting dude who live around the time of 500BC. He was a Greek philosopher, mathematician, and even started his own cult that believed numbers had mystical powers and beans contained the souls of the dead. He did lots of stuff, but his main claim to fame is Pythagoras' theorem, which links the three sides of a right-angled triangle together.

**Pythagoras' theorem**

$$a^2 + b^2 = c^2$$
$$c^2 - b^2 = a^2$$
$$c^2 - a^2 = b^2$$

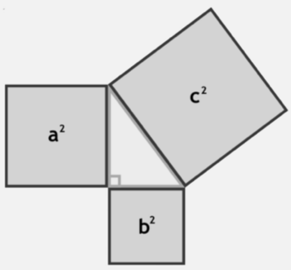



Where:

a & b = the two shorter sides of the triangle

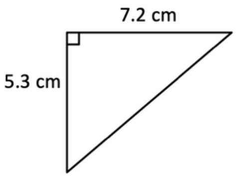
c = the hypotenuse (longest side, opposite the right angle)

It states: in a right-angled triangle, the square of the hypotenuse is equal to the square of the two other sides added together.



**Worked examples.**

Find the missing length in these right-angled triangles.

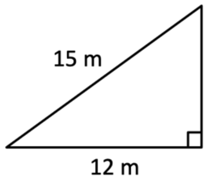



a = 5.3, b = 7.2, c = ?

$$5.3^2 + 7.2^2 = c^2$$

$$c = \sqrt{5.3^2 + 7.2^2}$$

$$c = 8.94cm$$



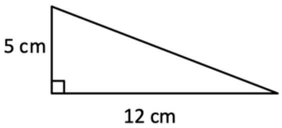

c = 15m, b = 12m, a = ?

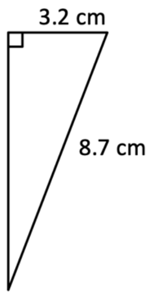
$$15^2 - 12^2 = a^2$$

$$a = \sqrt{15^2 - 12^2}$$

$$a = 9m$$

**With your teacher.**




a) A triangle has sides of 3cm, 4cm, and 5cm. Prove whether it is right-angled.



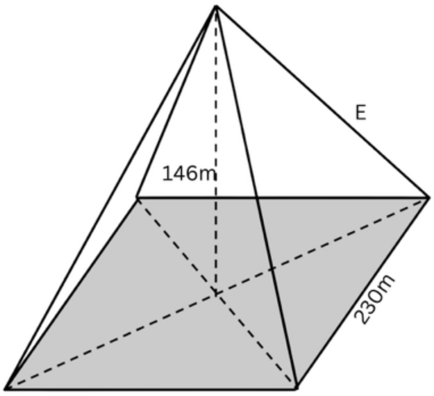
b) A ladder is leaning up against a wall. The bottom of the ladder is 2m away from the wall. The top of the ladder meets the wall at a point 5m off the ground. How long is the ladder?

**Key points:**

1. The hypotenuse is always the longest side (opposite the right angle).
2. You **add** the shorter side squares to find the hypotenuse
3. You **subtract** from the hypotenuse to find a shorter side.

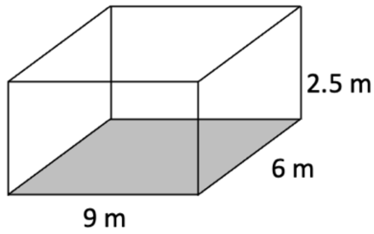
**With your teacher.**

1. The great pyramid of Khufu, in Egypt, was the world's tallest man-made structure for nearly 4,000 years. Standing at 146 metres tall, it has a square base with each side 230 metres long. The four corners of the pyramid face exactly North, South, East and West. Find the length of the sloping edge (E).



**With your teacher.**

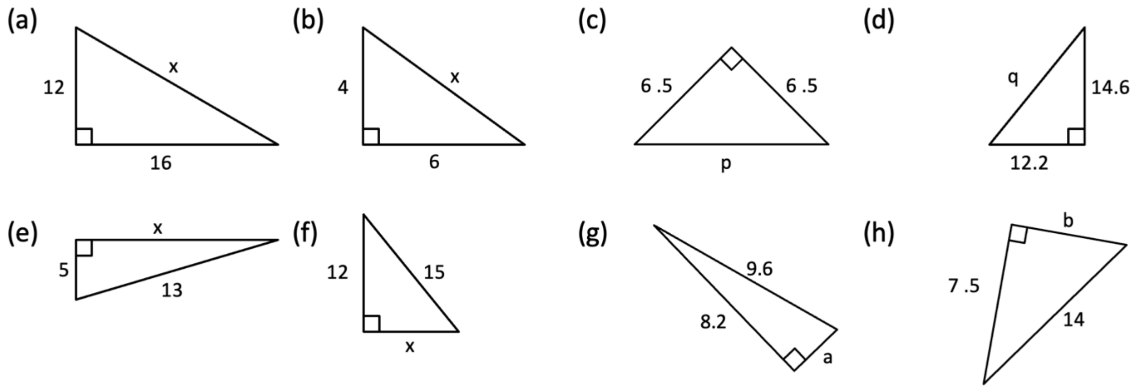
2. Find the length of the longest needle that fits into a box measuring 6m x 9m x 2.5m.




3. A light wind has blown a sky lantern 205m to the East and 180m to the North of its initial starting point. The straight-line distance to the top of the lantern is 620m. Find the vertical height of the lantern.

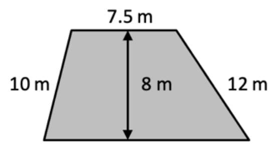
**Try it yourself (PYTHAGORAS).**

1. Use Pythagoras' Theorem to find the unknown side of each of the following triangles. (Measurements are in cm.)




2. A yacht sails 2.7 km due North and then 5 km due West. How far is it from its starting point?

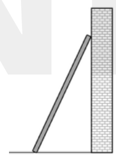
3. Find the perimeter of the trapezium shown.



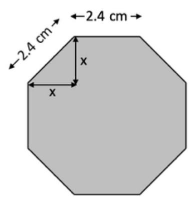

4. A rectangle is 10 cm in length. The distance from its centre to a vertex is 6.7 cm. Find the width of the rectangle.



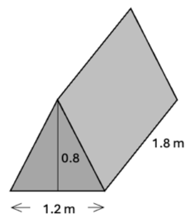
5. A ladder, 6.5 m in length, is resting with its feet 1.8 m from the base of a wall. The feet of the ladder slip a further 50cm from the wall. How far down the wall does the ladder slide?



6. Use Pythagoras' Theorem to find the dimension (x) of the octagonal tile shown (with each of the 8 sides measuring 2.4cm), then use it to find the area of the tile.



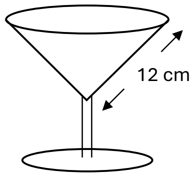

7. Find the area of nylon fabric used to make a tent which measures 1.2m wide, 1.8m long and 0.8m high. (The ends and sides are nylon, but different fabric is used for the floor.)



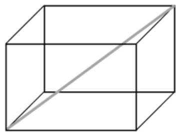

8. A lamp-shade has a sloping height of 40 cm, and upper and lower radii of 17 cm and 28cm. It sits on top of a stand 42cm in height. Find the total vertical height of the lamp.

**Try it yourself (PYTHAGORAS continued).**

9. A glass has a conical shape with a radius of 7 cm and a slant height of 12 cm.
   a) Find its volume. (Volume of cone = $^1/_3\,\pi r^2 h$ where h is the vertical height.)
   b) How many glasses would be needed to drink 1 litre?




10. Find the length of the longest needle that will fit into a rectangular box measuring 13cm x 6cm x 4cm.

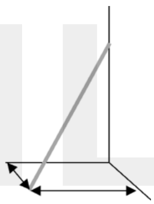


11. Find the distance connecting opposite diagonal corners of a box which measures 12cm x 4cm x 5cm.

12. Find the volume of a rectangular box if its base measures 7.5 cm x 12.6 cm and a diagonally opposite corners are separated by 22.7 cm

13. A submarine dives vertically to a depth of 1200 m before travelling 300m South and 720m East. How far from its starting point is it?

14. A broom stick 1.5 m in length leans in the corner of a room. The bottom of the stick is 0.5 m from one wall and 0.3 m from the other. How far up the wall does the broom stick reach?

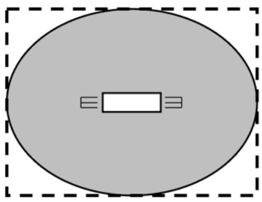


15. A glass pyramid serves as the main entrance for the Louvre in Paris. It has a vertical height of 21.6 m and a square base of side length 35 m. Find the area of glass used in its construction.

16. A kite is being flown on a string of length 220m. The kite is 80m East and 75m North of the flyer. Find the height of the kite.

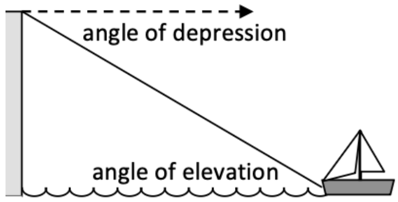
17. A drone with a camera is used to film a cricket match which is being played on an oval that measures 170m x 120m. The operator of the drone is instructed that he may not fly the drone inside the area indicated by the rectangle in the diagram. He also must not fly the drone at an altitude of lower that 50m. Find:

   a) The closest distance that the drone can get to the centre of the cricket pitch.
   b) The distance that separates the drone from the centre of the pitch when it is at the corner point of the rectangle.



## ELEVATION AND DEPRESSION

We often use trigonometry to solve problems about looking up or down at objects. This is known as elevation and depression.



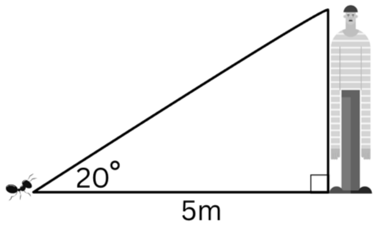

**Angle of Elevation**: Looking **up** from a horizontal.

**Angle of Depression**: Looking **down** from a horizontal.

Both angles are measured from a **horizontal line** (not from the object itself).

**Worked examples.**

1. An ant on the ground is looking up at the tallest man he has ever seen. The ant is 5 metres away from the man and is looking up at an angle of of 20°. How tall is the man?




$$\tan(20°) = \frac{x}{5}$$

$$x = 5 \times \tan(20°)$$

$$x = 1.82m$$

The man is 1.82 metres tall

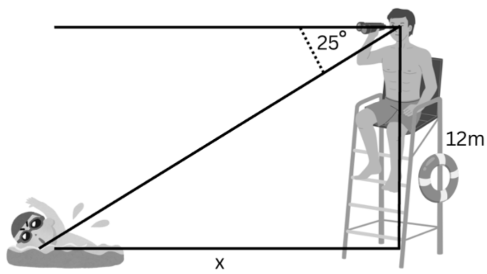
2. A lifeguard is sitting at the top of a lookout tower that is 12 m high. He spots a swimmer in the water at an angle of depression of 25°. How far is the swimmer from the base of the tower?

$$90° - 25° = 65°$$

$$\tan(65°) = \frac{x}{12}$$

$$x = 12 \times \tan(65°)$$

$$x = 25.73m$$



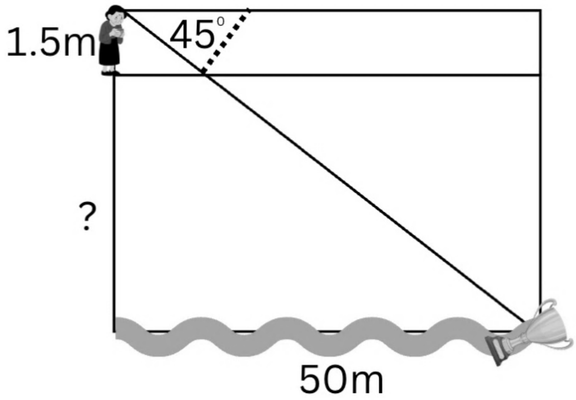

The swimmer is 25.73 metres from the base of the tower.

To use the trig ratios you need the angle **inside** a triangle.

Angles of depression often require an extra step to find the interior angle.

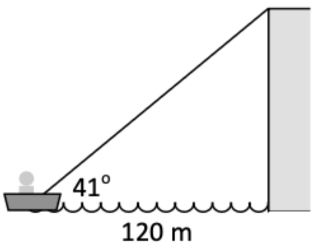
**With your teacher.**

1. Elizabeth Blackburn dropped her Nobel Prize trophy off a cliff near her childhood home in Launceston. Elizabeth contemplates jumping in after it but is not sure how high the cliff is. The trophy is currently floating 50m away from the base of the cliff, and Elizabeth is looking down at it at an angle of depression of 45° . If Elizabeth is 1.5 metres tall, how tall is the cliff?






2. A small boat 120m out to sea notes that the angle of elevation to of the top of a cliff is 41°. Find the height of the cliff.



**Worked example.**

A woman standing at position A finds that the angle of elevation to the top of a 220m high tower is 37°. Upon walking to position B, which is closer to the tower, she finds that the angle has become 59°. Find the distance that the woman walked between making the observations.

**Big triangle (A)**

$$\tan 37 = \frac{200}{A}$$

$$A = \frac{200}{\tan 37}$$

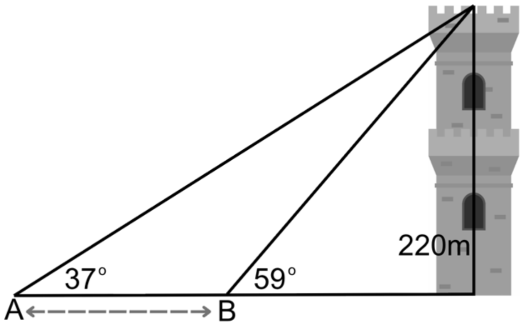
$$A = 265.41m$$

**Small triangle (B)**

$$\tan 59 = \frac{200}{B}$$

$$B = \frac{200}{\tan 59}$$

$$B = 120.17m$$

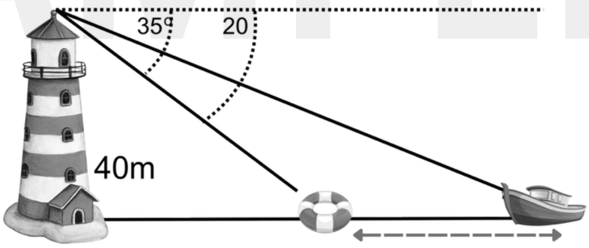



Distance between = 265.41 – 120.17 = 145.24

The woman walked 145.24 metres between observations.

**With your teacher.**

From the top of a lighthouse, 40m high, a lookout sees a small boat and a buoy in the same straight line out to sea. The angle of depression to the buoy is 35°. The angle of depression to the boat is 20°. Find how far apart the boat and the buoy are.



**Try it yourself (ELEVATION AND DEPRESSION).**

1. An observer on the top of a 50m high cliff notes that the angle of depression to a sea kayak is 60°. Find the distance of the sea kayak from the cliff.

2. The pilot of a light aircraft finds that the angle of depression to a landing strip is 12° and his altitude (height) is 250m. Find:
   a) The horizontal distance of the aircraft from the landing strip
   b) The straight-line distance of the aircraft from the landing strip

3. A lighthouse 50m high sits on top of a cliff. Some jagged rocks are located 120m out to sea from the base of the cliff and can be seen at an angle of depression of 68° when viewed from the top of the lighthouse. Find the height of the cliff.

4. An observer measures the angle of elevation of a tree to be 38° when viewed from a distance of 52m. Find the height of the tree allowing for the fact that the observers height to eye-level was 1.52m.

5. A surveyor notes that the angle of elevation of a building is 71° when viewed through a theodolite from a position 23m from the base of the building. Find the height of the building given that the surveyor's theodolite was 1.48m above ground level when the measurements were taken.



6. A lighthouse on the Tasman Peninsula is 30 m tall. A boat is anchored 50 m away from the base of the lighthouse. Find the angle of elevation from the boat to the top of the lighthouse.

7. A tourist is at the base of Mount Wellington (1271 m high). They are standing 2km horizontally from the summit. Find the angle of elevation to the top of the mountain.

8. A surveyor on Bruny Island sees the Cape Bruny Lighthouse. The angle of elevation to the top of the lighthouse is 12°, and the angle of elevation to the bottom is 8°. If the surveyor is 500 m away, calculate the height of the lighthouse.

9. A rescue helicopter hovers above Bass Strait at a height of 120 m. The crew spot a flare at an angle of depression of 18° and a life raft at an angle of depression of 25°. Both are in the same line of sight. How far apart are the flare and the raft on the water?

## PROBLEMS INVOLVING TWO TRIANGLES

More complicated trigonometry problems often involve multiple triangles.

The most important factor when dealing with multiple triangles is ensuring the diagram is clear and correct, and making a plan before you start.

**Worked example.**

A woman standing at position A finds that the angle of elevation to the top of a 220m high tower is 37°. Upon walking to position B, which is closer to the tower, she finds that the angle has become 59°.

Find the distance that the woman walked between making the observations.

**Big triangle (A)**

$$\tan 37 = \frac{200}{A}$$

$$A = \frac{200}{\tan 37}$$

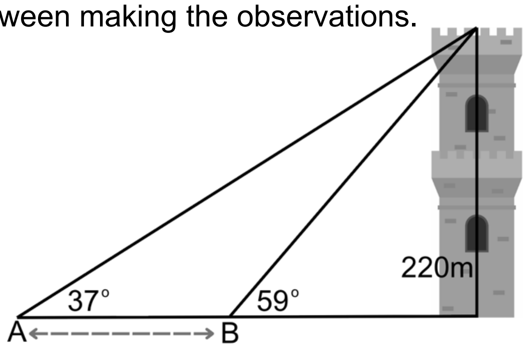
$$A = 265.41m$$

**Small triangle (B)**

$$\tan 59 = \frac{200}{B}$$

$$B = \frac{200}{\tan 59}$$

$$B = 120.17m$$



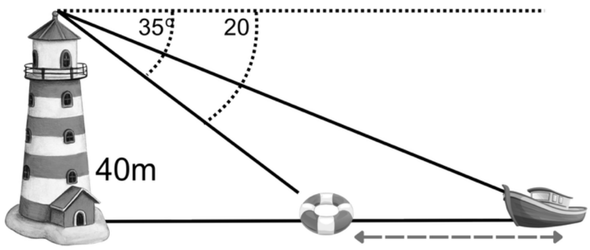

Distance between = 265.41 – 120.17 = 145.24

The woman walked 145.24 metres between observations.



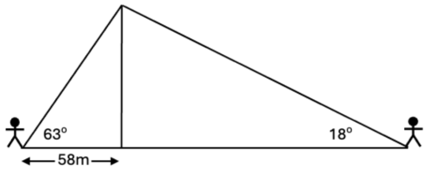
**With your teacher.**

From the top of a lighthouse, 40m high, a lookout sees a small boat and a buoy in the same straight line out to sea. The angle of depression to the buoy is 35°. The angle of depression to the boat is 20°. Find how far apart the boat and the buoy are.



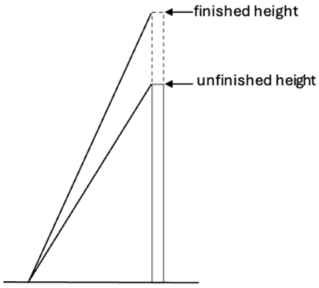
**Try it yourself (PROBLEMS INVOLVING TWO TRIANGLES)**

1. The angle of elevation of a tower is 63° when viewed by an observer who is 58 m from the base of the tower. When viewed by another observer who is on the other side of the tower - directly opposite the first observer the angle of elevation is 18°. Find the separation of the observers.

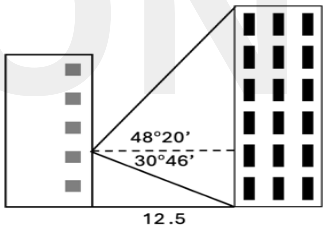



2. The angle of elevation of the top of a 425m high building is 37°. Find the angle of elevation if the observer walks 50m closer to the base of the building.

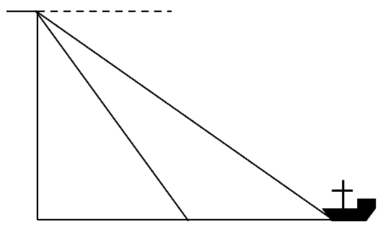
3. The angle of elevation to the top of an unfinished tower is 52° when viewed from an observation point 112 m from the base of the tower. When the tower is complete the angle of elevation to the towers top from the same observation point will be 76°. How much taller will the completed tower be?



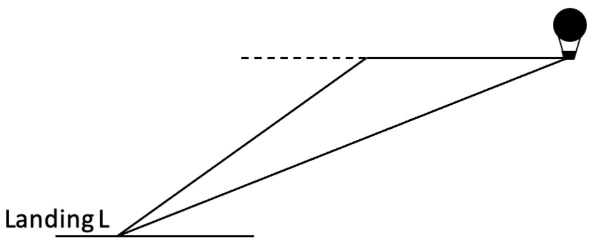

4. The angles of elevation and depression to the top and bottom of a building are 48°20' and 30°46' when viewed from part way up another building on the opposite side of the street. Find the height of the observed building if the street is 12.5m wide.




5. From the top of a 150m high cliff the angle of depression to a small boat at sea is 32°. After some time, the observer finds that the boat has moved closer to the cliff and the angle has become 62°. Find the distance that the boat has moved in between the observations.



6. The pilot of a hot air balloon which is drifting at altitude 800 m notes a suitable landing place with angle of depression 12°. Three minutes later the balloon is at the same altitude, but the angle of depression has become 18°. Find the drift speed of the balloon in km/h.



**Try it yourself (PROBLEMS INVOLVING TWO TRIANGLES cont)**

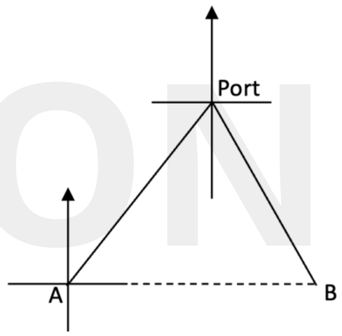
7. The angle of elevation to the top of an 85m tall building is 25° 15' when viewed by a man who is 1.8 m in height. Upon walking closer to the building, the man finds that the angle has become 49°54'. How far did the man walk in between making the observations?

8. A jet flies due North for a distance of 50 km and then on a bearing of N 70°E for a further 60 km. Find the distance and bearing of the jet from its starting point.

9. A ship sails on a bearing of S32°W for 50 km before heading due W for 80 km. Find its distance and bearing from its starting point.

10. A ship sailing at 15 km/h sails on a bearing of 030°T for 2 hours then due East for a further 1 hour and 20 mins. Find its distance and bearing from its starting point.

11. Two ships (A and B) set sail from the same port. Ship A sails on bearing S40°W and ship B on bearing S30°E. After sailing a distance of 5 km the captain of ship A notices that Ship B is due East of him. How far apart are the two ships?

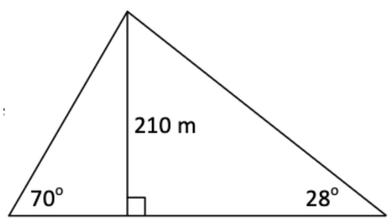





12. After walking for some time on a bearing of N30°W a bush walker notes that a mountain peak is due East of him. He knows that the peak is 7 km from the starting point on bearing N20°E. How far has the bush walker travelled?

13. After walking for 820 m upon an incline of 25° a bushwalker finds that the gradient of the hill increases to 35° for the final 300 m. Find the height of the hill.

14. The angle of elevation to the top of a 210 m high tower is 70° when viewed from by one observer. When viewed by another observer who is on the other side of the tower - directly opposite the first observer the angle of elevation is 28°. Find the separation of the observers.



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