2026

GENERAL MATHEMATICS

Level 3

TRIGONOMETRY

General Mathematics: Level 3

GM3 – TRIGONOMETRY

By Jess Bertram

With sincere thanks to John Short and Rick Smith.

ICON:	MEANING:
	Worked example
	Complete with your teacher
	Try it yourself
	CAS Calculator can be used
	Investigation

SAMPLE ONLY

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GENERAL MATHEMATICS - LEVEL 3

GM3 TRIGNOMETRY

BY JESS BERTRAM

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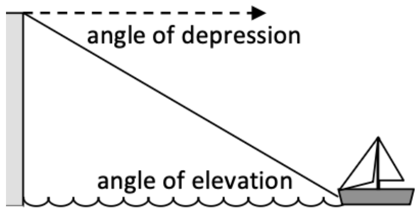
ELEVATION AND DEPRESSION

We often use trigonometry to solve problems about looking up or down at objects. This is known as elevation and depression.



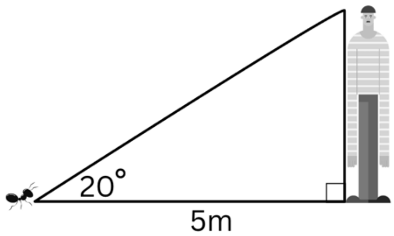
Angle of Elevation: Looking **up** from a horizontal.

Angle of Depression: Looking **down** from a horizontal.

Both angles are measured from a **horizontal line** (not from the object itself).

Worked examples.

1. An ant on the ground is looking up at the tallest man he has ever seen. The ant is 5 metres away from the man and is looking up at an angle of of 20°. How tall is the man?



$$\tan(20°) = \frac{x}{5}$$

$$x = 5 \times \tan(20°)$$

$$x = 1.82m$$

The man is 1.82 metres tall

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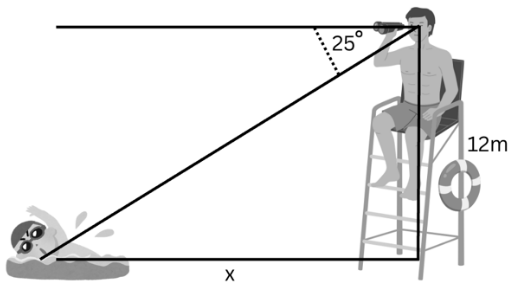
2. A lifeguard is sitting at the top of a lookout tower that is 12 m high. He spots a swimmer in the water at an angle of depression of 25°. How far is the swimmer from the base of the tower?



90° – 25° = 65°

$$\tan(65°) = \frac{x}{12}$$

$$x = 12 \times \tan(65°)$$

$$x = 25.73m$$

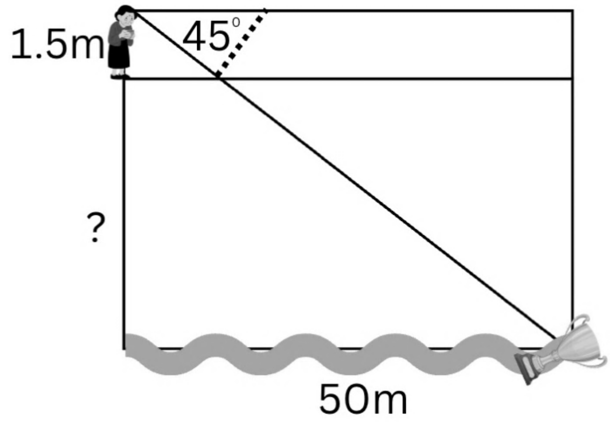
The swimmer is 25.73 metres from the base of the tower.

To use the trig ratios you need the angle **inside** a triangle.

Angles of depression often require an extra step to find the interior angle.

With your teacher.

1. Elizabeth Blackburn dropped her Nobel Prize trophy off a cliff near her childhood home in Launceston. Elizabeth contemplates jumping in after it but is not sure how high the cliff is. The trophy is currently floating 50m away from the base of the cliff, and Elizabeth is looking down at it at an angle of depression of 45^{o} . If Elizabeth is 1.5 metres tall, how tall is the cliff?



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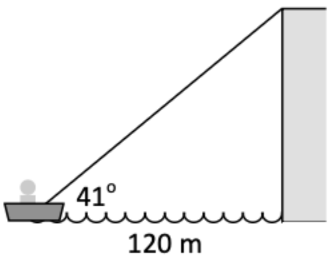
2. A small boat 120m out to sea notes that the angle of elevation to of the top of a cliff is 41^{o}. Find the height of the cliff.



Worked example.

A woman standing at position A finds that the angle of elevation to the top of a 220m high tower is 37°. Upon walking to position B, which is closer to the tower, she finds that the angle has become 59°. Find the distance that the woman walked between making the observations.

Big triangle (A)

$$\tan 37 = \frac{200}{A}$$

$$A = \frac{200}{\tan 37}$$

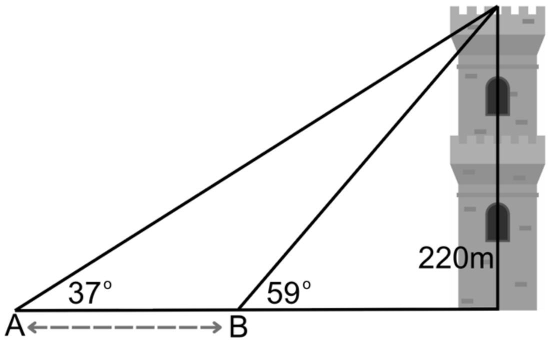
$$A = 265.41m$$

Small triangle (B)

$$\tan 59 = \frac{200}{B}$$

$$B = \frac{200}{\tan 59}$$

$$B = 120.17m$$



Distance between = 265.41 – 120.17 = 145.24

The woman walked 145.24 metres between observations.

With your teacher.

From the top of a lighthouse, 40m high, a lookout sees a small boat and a buoy in the same straight line out to sea. The angle of depression to the buoy is 35°. The angle of depression to the boat is 20°. Find how far apart the boat and the buoy are.

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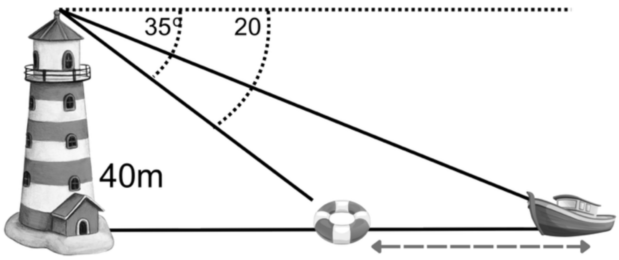
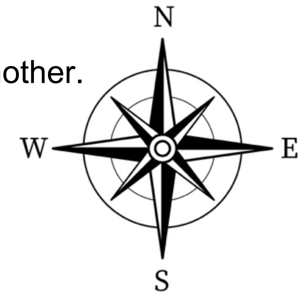


BEARINGS



Bearings are a way of describing the **direction** of one point from another.

There are two types of bearings covered in this workbook:

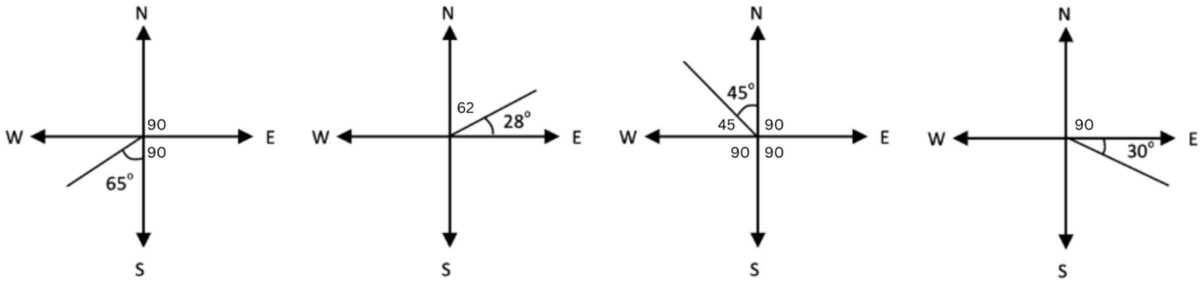
- True bearings
- Reduced bearings

True bearings

A true bearing is measured clockwise from North (0°), all the way around the circle.

It is always written as a three-digit number, followed by a ‘T’.

Some examples of a true bearing would be:



245°T   062°T   315°T   120°T

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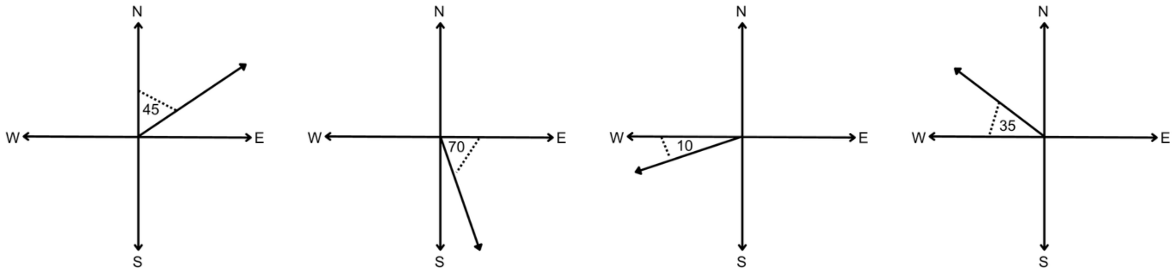
With your teacher.

1. Write the true bearings for the following diagrams.



2. A boat leaves the harbour and sails on a true bearing of 090° for 5 km. It then changes course and sails on a true bearing of 180° for 12 km. Draw a sketch of the boat’s journey and use Pythagoras’ theorem to find how far the boat is from the harbour.

Reduced bearings.

Reduced bearings describe a direction by measuring the smallest angle from either North or South.

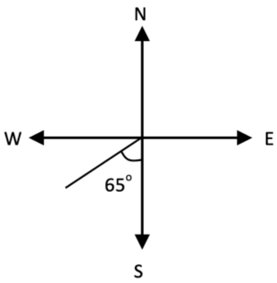
Reduced bearings start with N or S (N for North, S for South), then the angle (must be less than 90°) and then which side it is closest to (East or West).

S means start from South → **S65°W** ← *W means moving towards West*

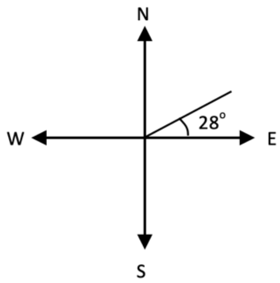
↑

65° means the angle from S towards W is 65°

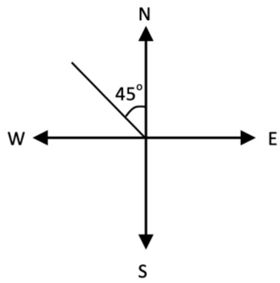
Examples of reduced bearings.



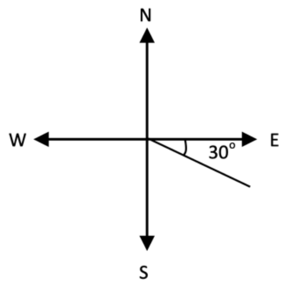
S65°W



N62°E



N45°W

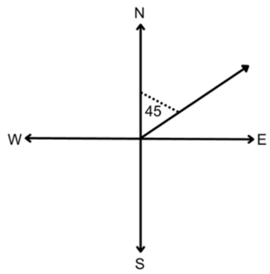


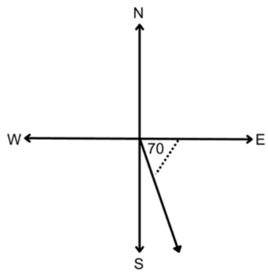
S60°E

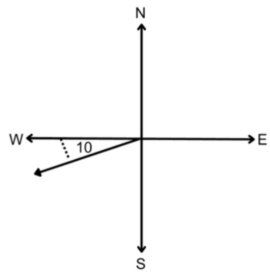
SAMPLE ONLY

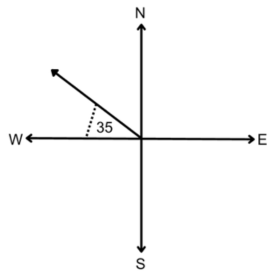
With your teacher.

1. Write the reduced bearings for the following diagrams.









2. A hiker leaves a campsite and walks 4 km on a bearing of N 60° E. She then turns and walks 9 km on a bearing of S 30° E. Draw a sketch of the hiker's journey and use trigonometric ratios or Pythagoras' theorem to calculate how far she is from the campsite.

PROBLEMS INVOLVING TWO TRIANGLES

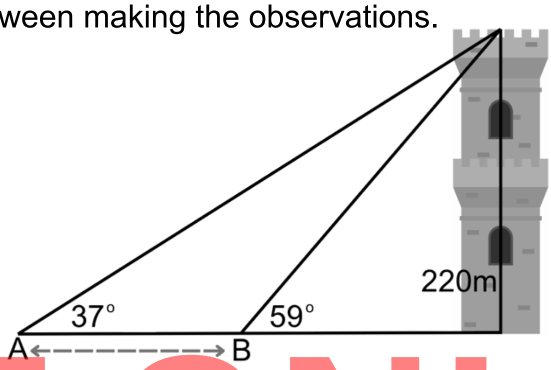
More complicated trigonometry problems often involve multiple triangles.

The most important factor when dealing with multiple triangles is ensuring the diagram is clear and correct, and making a plan before you start.

Worked example.

A woman standing at position A finds that the angle of elevation to the top of a 220m high tower is 37°. Upon walking to position B, which is closer to the tower, she finds that the angle has become 59°.

Find the distance that the woman walked between making the observations.



Big triangle (A)

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$$A = \frac{200}{\tan 37}$$

$$A = 265.41m$$

Small triangle (B)

$$\tan 59 = \frac{200}{B}$$

$$B = \frac{200}{\tan 59}$$

$$B = 120.17m$$

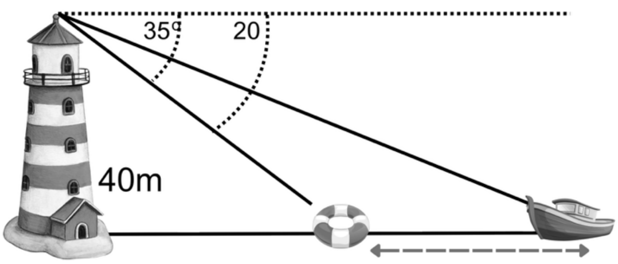
SAMPLE ONLY

Distance between = 265.41 – 120.17 = 145.24

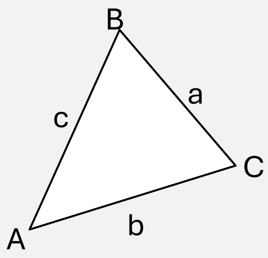
The woman walked 145.24 metres between observations.

With your teacher.

From the top of a lighthouse, 40m high, a lookout sees a small boat and a buoy in the same straight line out to sea. The angle of depression to the buoy is 35°. The angle of depression to the boat is 20°. Find how far apart the boat and the buoy are.



The cosine rule (for any triangle)



$$a^2 = b^2 + c^2 - 2bc\cos A$$ To find an unknown side

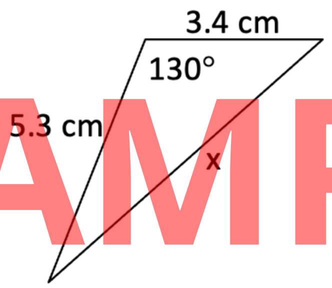
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$ To find an unknown angle

The Cosine Rule can be used for problems which involve 3 sides and 1 angle.

The two known sides must form the angle.

Worked examples.

1. Find the unknown side (x) in this triangle.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$x^2 = 5.3^2 + 3.4^2 - 2(5.3)(3.4)\cos 130°$$

$$x^2 = 62.82$$

$$x = 7.93\text{cm}$$

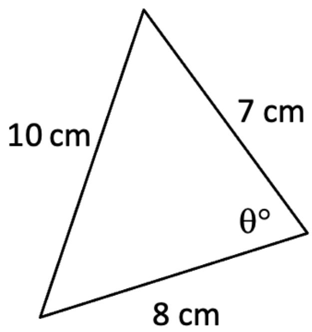
SAMPLE ONLY

Tips for success:

Enter the whole equation into your calculator in one go. This avoids losing accuracy due to rounding.

Ensure your calculator is in degrees mode (not radians)

2. Find the unknown angle (θ) in this triangle.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

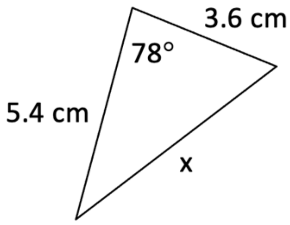
$$\cos\theta = \frac{7^2 + 8^2 - 10^2}{2(7)(8)}$$

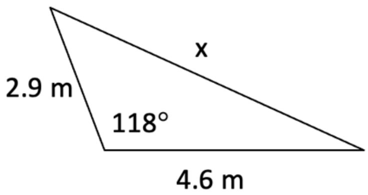
$$\theta = 83.33°$$

$$\theta = 83°20'$$

With your teacher.

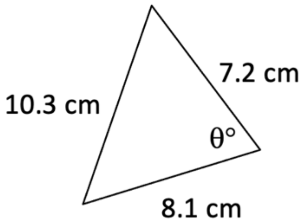
1. Find the unknown side (x) in each of the following triangles.





SAMPLE ONLY

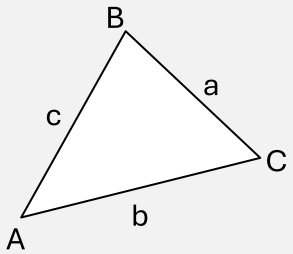
2. Find the unknown angle in the following triangle.



THE SINE RULE

The sine rule can be used to find a missing angle or side in **any** triangle.

The Sine rule

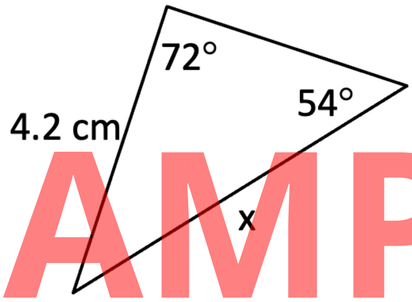


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ To find an unknown side

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$ To find an unknown angle

Worked examples.

1. Find the unknown side (x) in the below triangle.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

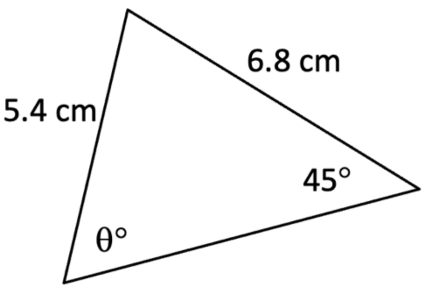
$$\frac{x}{\sin 72°} = \frac{4.2}{\sin 54°}$$

$$x = \frac{4.2}{\sin 54°} \times \sin 72°$$ ← Multiply both sides by $\sin 72°$ to get x by itself.

$$x = 4.94\text{cm}$$

SAMPLE ONLY

2. Find the unknown angle (θ) in the below triangle.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{6.8} = \frac{\sin 45°}{5.4}$$

$$\sin \theta = \frac{\sin 45°}{5.4} \times 6.8$$ ← Multiply both sides by 6.8

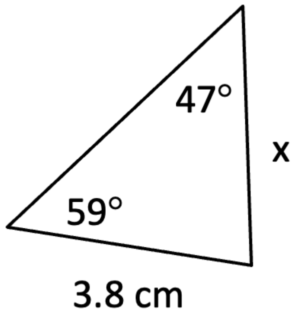
$$\theta = \sin^{-1}(\frac{\sin 45°}{5.4} \times 6.8)$$ ← Find inverse of Sine

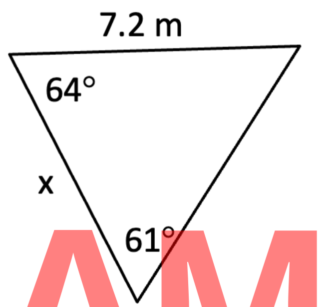
$$\theta = 62.93°$$

$$\theta = 62°56'$$

With your teacher.

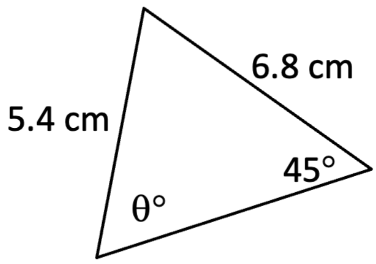
1. Find the unknown sides (x) in each of the following.



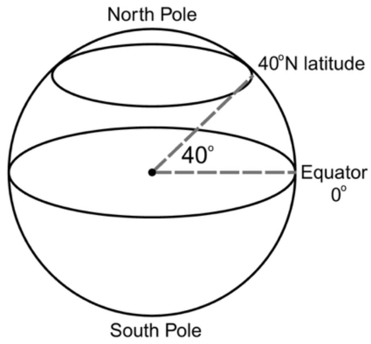


SAMPLE ONLY

2. Find the unknown angle (θ) in the following triangle.



LATITUDE



Lines of latitude (also known as 'parallels of latitude') are:

- Imaginary circles on Earth's surface.
- Parallel with the equator.
- Tell you how far North or South you are of the Equator.
- All different sized circles. The biggest circle is on the equator, then they get smaller the further they get towards the North and South poles.

There are 180 degrees of latitude in total (90°N + 90°S), with about 111km between each degree of latitude (a bit further than Launceston to Devonport). But this can be further broken up into degrees, minutes and seconds.

***Tip:** people often remember latitude as FLATitude. The lines are flat or horizontal.*

With your teacher.

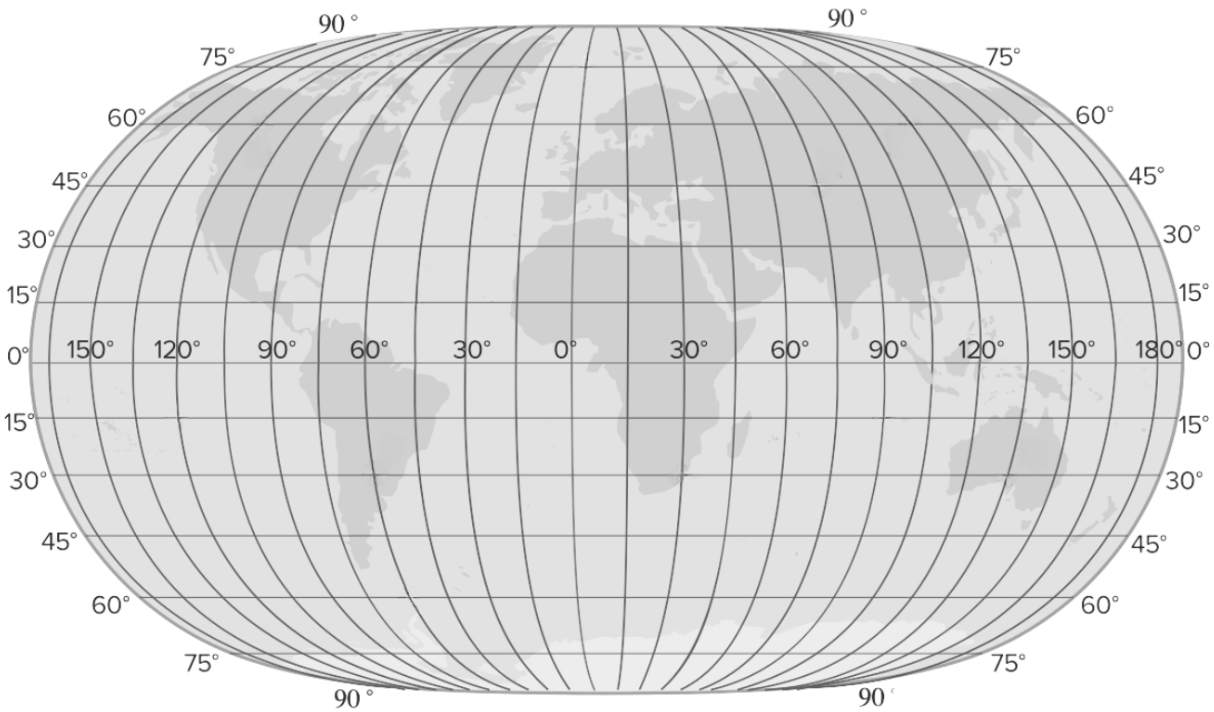
Using the diagram below, find an approximate latitude (to the nearest 10°) for the following locations:

SAMPLE ONLY

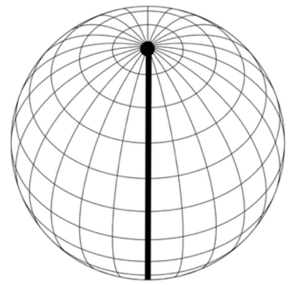
a. Tasmania　　b. North Pole　　c. Madagascar

d. Southern Italy　　e. Florida USA　　f. London, England

g. The southernmost tip of South America　　h. Auckland NZ



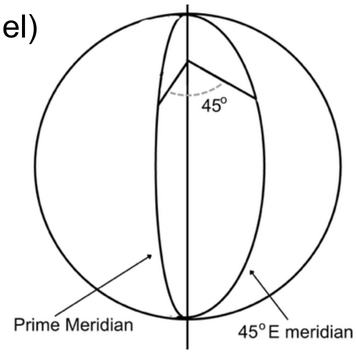
LONGITUDE



Prime Meridian
0°

Lines of longitude (sometimes called 'meridians') are:

- Imaginary circles on the Earth's surface
- Each circle is the same size, and has the same radius as the Earth (≈ 6371 km)
- All lines intersect at the North and South poles (not parallel)
- Lines of longitude tell you how far East or West you are of the Prime Meridian.
- Each meridian is labelled as a number of degrees east or west of the Prime Meridian (0°).
- Meridians can go all the way up to 180°E or 180°W.



What is the Prime Meridian?

The Prime Meridian is the line of longitude that passes through Greenwich and represents 0 degrees East or West. The Prime Meridian separates the Eastern and Western Hemispheres. If you were to stand on the Prime Meridian with one foot either side, you would be both in the East and the West at the same time.

SAMPLE ONLY

Why Greenwich?

Prior to the International Meridian Conference in 1884, many countries had different navigation systems based on big cities, popular ports, or even significant religious sites. Life got very confusing and a little dangerous when you sailed from one country to the next. Greenwich, on the bank of River Thames in England, had a strong maritime history, many widely used maps, and eventually won the popular vote at the conference. Or England bribed someone. We will never know.

With your teacher.

1. Using the map on the previous page, find the longitude (to the nearest 10°) for:

   a. Tasmania b. North Pole c. Madagascar

   d. Southern Italy e. Florida USA f. London, England

2. When giving a location you give the latitude first, then the longitude. Write the full location (to the nearest 10°) for each of the places from question 1.

GPS CO-ORDINATES

Locations can be written in different formats, such as the format used by GPS devices and apps such as Maps and Google Earth.

Some minor differences include:

- Use decimal degrees (DD) format instead of degrees minutes seconds (DMS)
- Latitudes South of the equator are represented by negative numbers
- Longitudes West of the Prime Meridian are represented by negative numbers
- The location is written without the use of the initials N, S, E or W.

Worked example.

The Devonport *'Spirit of Tasmania'* terminal is located at 41° 10' 46"S, 146° 21' 56"E

Give this location using GPS co-ordinates.

Step 1: Convert degrees minutes seconds (DMS) into decimal degrees (DD). This can be done with your calculator **or** by hand.

SAMPLE ONLY

Convert DMS to DD by hand.

DMS = 41° 10' 46"S (Latitude)

DD = D + M/60 + S/3600

DD = 41 + 10/60 + 46/3600

DD = 41.179444°

Convert DMS to DD using CASIO Classpad

DMS = 146° 21' 56"E (Longitude)

On the main screen of your calculator go to

Interactive → Transformation → DMS → dms

Enter the degrees, minutes and seconds

Press Ok.

DD = 146.36555°

Step 2: Change any South or West locations into negatives.

The latitude is **south** of the equator, so will be negative.

The longitude is **east** of the Prime Meridian, so will be positive.

Final answer: – 41.17944°, 146.3655°

Worked example.

The Statue of Liberty is located at 40.68988, –74.04554.

Give the latitude and longitude of this landmark using degrees, minutes and seconds (DMS).

Step 1: Convert decimal degrees (DD) into degrees minutes seconds (DMS).

This can be done with your calculator **or** by hand.

Convert DD to DMS using CASIO Classpad

DD = 40.68988 (Latitude)

On the main screen of your calculator go to

Interactive → Transformation → DMS → toDMS

Enter 40.68988

Press Ok.

DMS = 40° 41’ 24”

SAMPLE ONLY

Convert DD to DMS by hand.

DD = –74.04554 (Longitude)

D = 74.04554 — The whole number (74) stays as degrees.

M = 0.04554 × 60 = 2.7324 — Multiply the decimal (0.04554) by 60. Only use the whole number as the minutes.

S = 0.7324 × 60 = 44 — Multiply the decimal from the minutes (0.7324) by 60 to find the seconds.

DMS = 74° 02’ 44”

Step 2: Change positives into North or East, and the negatives into South or West

The latitude is **positive**, so will be **North**.

The longitude is **negative**, so will be **West.**

Final answer: 40° 41’ 24” N, 74° 02’ 44” W

GM3 Trigonometry

2026 version

Jess Bertram

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