2026

GENERAL MATHEMATICS

Level 3

TEACHER EDITION

SEQUENCES

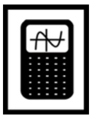
TEACHER EDITION

General Mathematics: Level 3

GM3 – SEQUENCES

By Jess Bertram

With sincere thanks to John Short and Rick Smith.

ICON:	MEANING:
	Worked example
	Complete with your teacher
	Try it yourself
	CAS Calculator can be used
	Investigation

For ordering, feedback and requests please visit:

BertramEducation.com

Teachers Edition.

This Teacher's Edition includes features not included in the student workbooks. Anything not in the student workbooks is printed in **red**.

PLANNER – SEQUENCES

Use the planner below to schedule lessons or track progress.

Use the order of the book or make your own. It's up to you.

BERTRAM
EDUCATION

Date	Content	Notes
	Intro to sequences Pages: 1, 2	
	Arithmetic sequences (A) Pages: 3, 4, 5, 6	
	A - Tables and graphs Pages: 7, 8, 9, 10, 11	
	A - Nth term rule Pages: 12 → 20	
	A - Sum of terms (series) Pages: 21 → 27	
	Geometric sequences (G) Pages: 28, 29, 30, 31	

TASC CURRICULUM DOCUMENT

Topic 2 - growth and decay in sequences

Key knowledge and skills

The arithmetic sequence

- use recursion to generate an arithmetic sequence
- represent terms of an arithmetic sequence in both tabular and graphical form
- use arithmetic sequences to model linear growth and decay in discrete situations
- deduce a rule for the nth term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, $t_n = a + (n-1)d$ and use this rule to make predictions
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay
- determine the sum, to n terms, of an arithmetic sequence, represented as $S_n = \frac{n}{2}(a + l)$ or $S_n = \frac{n}{2}(2a + (n-1)d)$

The geometric sequence

- use recursion to generate a geometric sequence
- represent the terms of a geometric sequence in both tabular and graphical form
- use geometric sequences to model exponential growth and decay in discrete situations
- deduce a rule for the nth term of a particular geometric sequence from the pattern of the terms in the sequence, $t_n = ar^{n-1}$ and use this rule to make predictions
- use geometric sequences to model and analyse, numerically, or graphically only, practical problems involving geometric growth and decay
- determine the sum, to n terms, of an arithmetic sequence, represented as $S_n = \frac{a(1-r^n)}{1-r}$, where $r \neq 1$.
- determine the sum of an infinite geometric series, represented as $S_\infty = a_1/(1-r)$

Sequences generated by first-order linear recurrence relations

- use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form
- recognise that a sequence generated by a first-order linear recurrence relation can have a long-term increasing, decreasing or steady-state solution
- use first-order linear recurrence relations to model and analyse, numerically or graphically only, practical problems involving growth or decay, with or without the aid of technology.

With your teacher.

A radioactive sample decays by 12 grams every hour. At the start of the monitoring period the sample has 100 grams of material remaining.

a. Represent this situation using a sequence. Write the first 6 terms ($t_1 \rightarrow t_6$)

$100,\ 88,\ 76,\ 64,\ 52,\ 40$

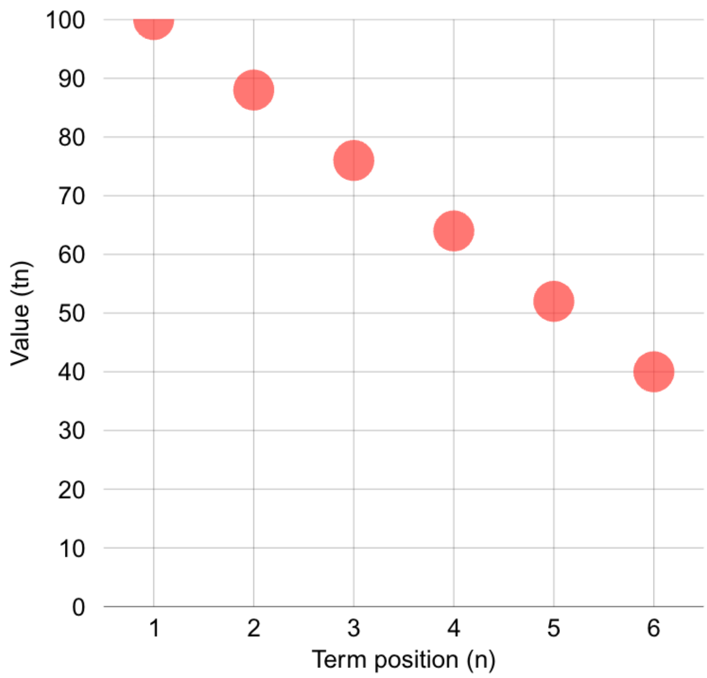
b. Identify the first term (t_1) and common difference (d).

First term $= 100$, Common difference $= -12$

c. Fill in the below table.

Term position (n)	Value (t_n)		Point @ (n, t_n)
1	100	→	(1, 100)
2	88	→	(2, 88)
3	76	→	(3, 76)
4	64	→	(4, 64)
5	52	→	(5, 52)
6	40	→	(6, 40)

d. Graph your points.



e. If we look at the 10th term, the sample has a weight of -8 grams. Is this possible in real life? Discuss.

No, not possible for the sample to have a negative weight.

Good opportunity to facilitate gentle discussion around limits, realistic limitations of models, even interpolation and extrapolation.

With your teacher.

1. The first term of an arithmetic sequence is 14. The 7th term is 50. Find:
   a. The common difference 'd' and first term 'a'.
   b. The rule for the sequence.

To find 'd'

$$\begin{aligned} t_1 + 6d &= t_7 \\ 14 + 6d &= 50 \\ 6d &= 36 \\ d &= 6 \end{aligned}$$

← The 1st term, plus 6 lots of the common difference, will equal the 7th term.

'a' is given = 14.

Rule for the sequence: $t_n = a + (n-1)d$

$t_n = 14 + (n-1)6$ ← *nth term rule for the sequence*

$t_n = 6n + 8$ ← *Simplified nth term rule*

2. The 6th term of an arithmetic sequence is 123. The 11th term is 98. Find;
   a. The common difference 'd' and first term 'a'.
   b. The rule for the sequence.
   c. The 16th term.

To find 'd'

$$\begin{aligned} t_6 + 5d &= t_{11} \\ 123 + 5d &= 98 \\ 5d &= -25 \\ d &= -5 \end{aligned}$$

← *The 6th term, plus 5 lots of the common difference, will equal the 11th term.*

To find 'a'

$$\begin{aligned} a + (6-1)(-5) &= 123 \\ a + (-25) &= 123 \\ a &= 148 \end{aligned}$$

← *This has been done with the 6th term (123), but we could use the 11th term (98) instead.*

b) The rule for the sequence.

$t_n = a + (n-1)d$

$t_n = 148 + (n-1)(-5)$ ← *Nth term rule for the sequence*

$t_n = -5n + 153$ ← *Simplified Nth term rule*

c) Find the 16th term.

$t_{16} = 148 + (16-1)(-5)$

$t_{31} = 73$ ← *16th term is 73*

With your teacher.

2. Find the sum of the sequence: 2000, 1000, 500, 250, … to eight terms.

$Common\ ratio\ (r) = 0.5$

$First\ term\ (a) = 2{,}000$

$Number\ of\ terms\ (n) = 8$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{2{,}000(1-0.5^8)}{1-0.5}$$

$$\boldsymbol{S_8 = 3{,}984.38}$$

3. Vashti has a coffee cart. On her first day of business she sells 100 cups of coffee. On her second day she sells 120 cups of coffee. She hopes that this is the start of a geometric sequence! If Vashti 's hopes are right find:

   a. The first 5 terms of the sequence. (Round to the nearest whole cup)

      100, 120, 144, 173, 207

   b. Find the total sold over the first 10 days.

      $Common\ ratio\ (r) = 1.2$

      $First\ term\ (a) = 100$

      $Number\ of\ terms\ (n) = 10$

      $$S_n = \frac{a(1-r^n)}{1-r}$$

      $$S_{10} = \frac{100(1-1.2^{10})}{1-1.2}$$

      $$\boldsymbol{S_{10} = 2{,}595.87}$$

      Vashti would sell 2,596 cups of coffee over the first 10 days.

   c. If this pattern continues, how many days will it be before Vashti sells 743 cups of coffee? (Hint: Find the sequence rule, then use your calculator to solve for n).

      $t_n = a \times r^{n-1}$

      $t_n = 100 \times 1.2^{n-1}$ ← Sequence rule

      $743 = 100 \times 1.2^{n-1}$

      Casio Classpad:

      Action → Advanced → solve

      Solve$(743 = 100 \times 1.2^{x-1},\ x)$ → EXE

      $x = 11.999$

      Vashti will first reach 743 cups of coffee sold on day 12.

With your teacher.

4. Write down the first 5 terms of the sequences given by the following difference equations;

   a. $t_{n+1} = 2t_n \qquad t_1 = 5$

      $5, \quad 10, \quad 20, \quad 40, \quad 80$

   b. $t_{n+1} = 0.5t_n \qquad t_1 = 1{,}000$

      $1000, \quad 500, \quad 250, \quad 125, \quad 62.5$

   c. $t_{n+1} = -5t_n \qquad t_1 = 6$

      $6, \quad -30, \quad 150, \quad -750, \quad -3750$

5. Write down the rule for generating the following sequences in recursive form (i.e. using a difference equation.)

   a. 4, 12, 36, 108, … $r = t_2 \div t_1 = 12 \div 4 = 3$

      $t_{n+1} = rt_n \qquad t_1 = a$

      $\boldsymbol{t_{n+1} = 3t_n \qquad t_1 = 4}$

   b. 128, 64, 32, 16, … $r = t_2 \div t_1 = 64 \div 128 = 0.5$

      $t_{n+1} = rt_n \qquad t_1 = a$

      $\boldsymbol{t_{n+1} = 0.5t_n \qquad t_1 = 128}$

6. The n^th term of a geometric sequence is given by the rule: $t_n = 2 \times 5^{n-1}$ Detail the first 5 terms of the sequence then give its rule in the form of a difference equation.

   First term $t_1 = 2 \times 5^{1-1} = 2$

   Second term $t_2 = 2 \times 5^{2-1} = 10$

   Third term $t_3 = 2 \times 5^{3-1} = 50$

   Fourth term $t_4 = 2 \times 5^{4-1} = 250$

   Fifth term $t_5 = 2 \times 5^{5-1} = 1{,}250$

   First 5 terms: 2, 10, 50, 250, 1250

   $r = t_2 \div t_1 = 10 \div 2 = 5$

   $t_{n+1} = rt_n \qquad t_1 = a$

   $\boldsymbol{t_{n+1} = 5t_n \qquad t_1 = 2}$