

2026

GENERAL MATHEMATICS

Level 3

NETWORKS

General Mathematics: Level 3

GM3 - NETWORKS

By Jess Bertram

With sincere thanks to John Short and Rick Smith.

ICON:	MEANING:
	Worked example
	Complete with your teacher
	Try it yourself
	CAS Calculator can be used
	Tips / shortcuts

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GENERAL MATHEMATICS - LEVEL 3

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INTRODUCTION TO GRAPHS

A **graph** is a diagram that is used to represent a network.

The graph of a network consists of a set of points called **vertices**, which are connected by lines called **edges**.

The edges represent the connections or relationships between the vertices.



Example.

The Bertram siblings (Alex, Brad, Chris, Darrah, Ebony, Fallon, George and Hayley) have signed up for SnapChat.

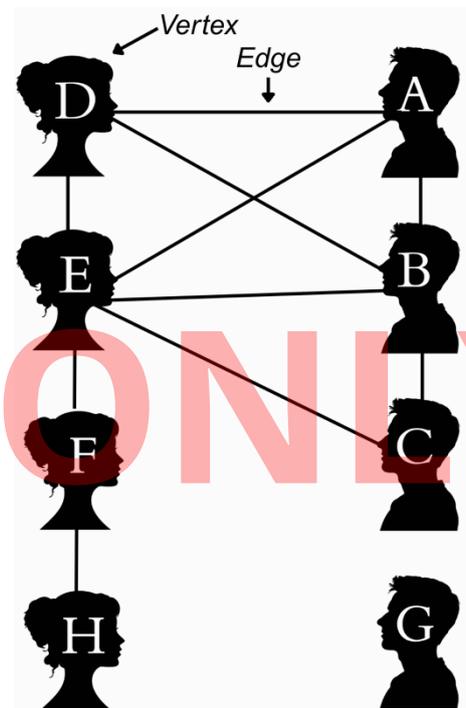
The **graph** to the right shows the connection between each sibling.

Each sibling is represented by a **vertex**.

The connection between each sibling is represented by a line called an **edge**.

The line between Darrah (D) and Alex (A) shows that they are friends on SnapChat. Darrah is also friends with Brad (B) and Ebony (E).

Darrah has 3 connections total, so the **degree of the vertex** is 3.



With your teacher.

1. State the number of connections each sibling has. (degree of each vertex)

A= B= C= D= E= F= G= H=

2. How many total connections (edges) are there between all siblings?

3. Is George friends with any of his siblings on SnapChat?

4. Who is the most popular sibling? (Which vertex has the highest degree?)

5. How many vertices are there in the graph? What do they represent?

In the previous example, there were no arrows on the ends of the edges, so it is assumed the edges go both ways. Darrah is friends with Alex on SnapChat, and Alex is also friends with Darrah.

When an edge only goes in one direction it is represented by an arrow and called an **arc**. A graph with arcs in it is known as a **directed graph** or **digraph**.



Example.

The Bertram's are off to a dance. Their mother is trying to marry them off but can't keep track of who likes who. She knows two girls (Molly and Piper) like her son Chris, but Chris only likes Molly.

She draws a **directed graph** or **digraph** to represent the current situation.



Her daughter Darrah is proving quite popular and has a few marriage prospects.

Some Darrah likes, some she doesn't.

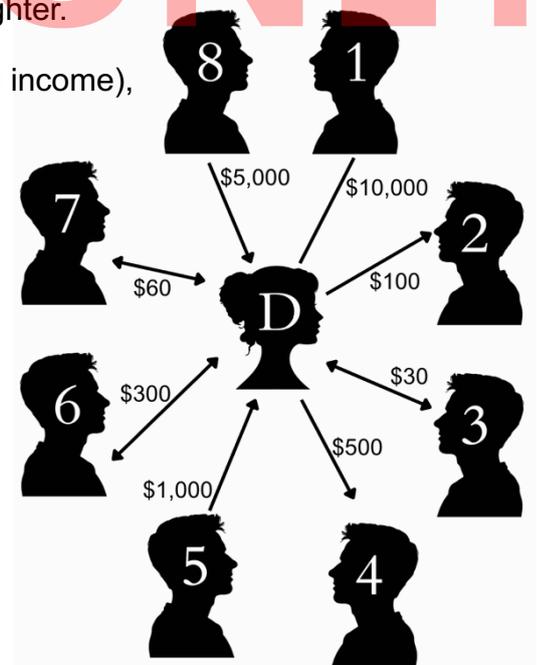
Mumma Bertram draws a network digraph showing each potential match and their annual income to help make the decision with her daughter.

When the edges are assigned weights (such as annual income), the graph is called a **weighted graph**.



With your teacher.

1. How many men in the digraph is Darrah interested in marrying?
2. How many men are interested in marrying Darrah?
3. If Darrah wants to marry someone she likes, but also wants them to make over \$100 a year, who should she choose?





With your teacher.

1. Construct a graph that could represent the following networks.
 - a. Four current tennis champions meet in a 'round robin' tournament. (In a round robin, everyone plays everyone else).

- b. A network of roads connects five towns:

Roads connect Southbridge directly to Franklin, Everton and Bell.

There are also roads from Franklin to Everton and from Everton to Bell.

There is an alternative direct route from Bell and Franklin that does not pass through Everton.

A road connects Bell to Colt Ridge.

SAMPLE ONLY

- c. Alan, Barry, Carrie are all friends. Alan also is friends with David and Elsa. Barry is friends with Elsa but does not know David.
 - i. Draw a graph of the original friendship network.
 - ii. How could you adjust the friendship network if Barry considers himself a friend of Carrie, but Carrie does not consider herself a friend of Barry?



Try it yourself (INTRO TO GRAPHS) *Answers page 77*

1. Draw a network graph for each of the following.

a. Network 1.

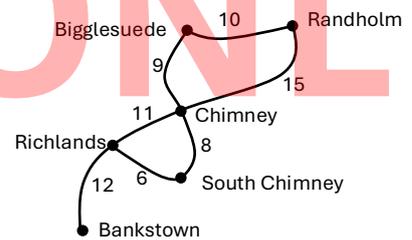
- A is connected with B and C.
- B and C are also connected to D.
- D is also connected to E.

b. Network 2.

- A is connected to B.
- B is connected to C.
- C is connected to D.
- D is connected to A.
- A and C are connected.
- B and D are connected.
- D is connected to E.

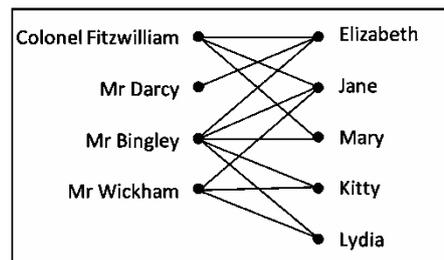
2. The graph shows a road network connecting towns.

- a. How many edges and how many vertices are used to draw the graph?
- b. Find the distance between Bankstown and Randholm via South Chimney and Bigglesuede.
- c. Find the distance between Bankstown and Randholm via the shortest possible route.



3. The network shows; 'who danced with who'.

- a. Who did Elizabeth dance with?
- b. Who danced with the most partners?
- c. Who danced with the least?



4. Construct a graph to represent the following friendship network:

- Jenny is friends with Kate, Lenny and Maddy
- Kate also counts Maddy as a friend but does not know Lenny. She is the only one to be a friend of Neil.
- Maddy , Lenny and Oli are all friends.



Try it yourself (INTRO TO GRAPHS cont) *Ans pg77*

5. Construct a directed graph to represent a food web which includes the following: Kookaburra, mouse, grass, garden vegetables, possum, rabbit, dingo.

Why is a directed graph appropriate in this instance?

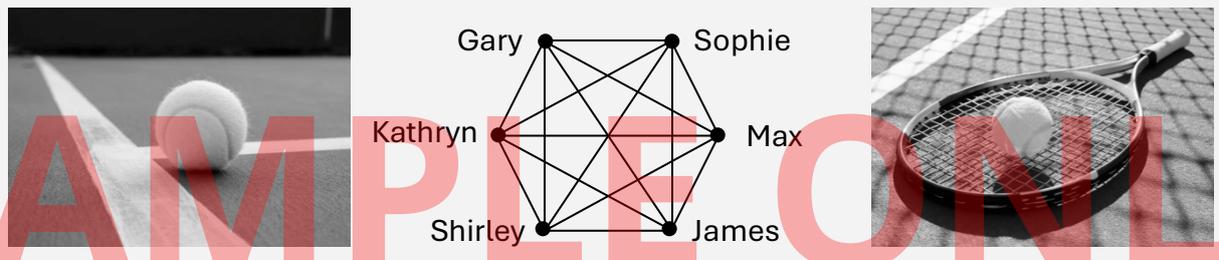
6. Consider the states of Australia (including ACT and NT).

a. Construct a graph in which the vertices are the states. Join vertices with an edge if states share a common border.

b. Which state is 'most connected'?



7. A tennis club organises a 'round robin' competition between six players. Each player plays one game against every other player.



a. What are the vertices of the graph?

b. What do the edges represent?

c. What is the significance of the degree of each Vertex?

d. How many matches are played in total?

e. Redraw the graph as a directed graph given the information that:

- Kathryn won 5 matches.
- Max beat Gary.
- James won 4 matches
- Sophie won 2 matches but lost to Max.
- Shirley lost all her matches.

f. Order the six players.

